



Lecture 09

Statistics II

STAT-221 • UG

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- **1. The Breakdown of Multiple t -Tests:**
 - ▶ Type I Error Inflation.
 - ▶ Why We Need Analysis of Variance (ANOVA).
- **2. The F -Distribution (Chapter 12):**
 - ▶ Four Key Characteristics of the F -Distribution.
 - ▶ Reading F -Tables using Numerator (df_1) and Denominator (df_2).
- **3. Testing Two Population Variances:**
 - ▶ Example 1: Lammers Limos (Driving Route Consistency).
 - ▶ Example 2: Google Page Speed (Website Redesign Load Times).
- **4. Introduction to One-Way ANOVA:**
 - ▶ Assumptions of One-Way ANOVA.
 - ▶ Partitioning Variance: Between vs. Within Treatments.
 - ▶ Constructing the ANOVA Table.
 - ▶ Example 3: Airline Passenger Ratings.
 - ▶ Software Application (Excel ANOVA Output).



Post-Midterm Roadmap

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Today's Learning Objectives (CLO 3, CLO 4):

- Understand the mathematical characteristics of the F -distribution.
- Conduct a hypothesis test comparing two population variances.
- Execute a One-Way Analysis of Variance (ANOVA) to compare three or more population means.



Discussion

Google Page Speed

A popular women's clothing retailer is redesigning its website to improve image loading times.

- The new layout is faster on average, but initial tests suggest its load times are much more **variable** across different web browsers.
- Management wants to know: Is the *consistency* of the new page significantly worse than the old page?

Why is measuring variability/consistency just as important as measuring the average speed in business?



Warm-up: The Problem with Multiple t -Tests

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- To compare 4 locations, we would need to conduct **6 separate t -tests**:

$$\text{Number of pairs} = \frac{k(k-1)}{2} = \frac{4(3)}{2} = 6 \text{ pairs}$$

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Type I Error Inflation!

If each test has a significance level of $\alpha = 0.05$, the probability of making **at least one** false positive error across 6 tests accumulates rapidly!



Mathematical Proof of Error Build-Up

If the significance level for a single test is $\alpha = 0.05$, the probability of correctly *not* rejecting a true null hypothesis is $1 - 0.05 = 0.95$.



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Instead of a 5% risk of error, running 6 separate t -tests gives us a **26.5% chance** of falsely finding a difference! **ANOVA solves this by comparing all means in a single test.**



The F -Distribution

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Four Core Characteristics of the F -Distribution:

- 1 **Continuous:** F can take an infinite number of values between 0 and $+\infty$.
- 2 **Non-Negative:** F values can **never be negative** because variances are squared numbers ($s^2 \geq 0$).
- 3 **Positively Skewed:** The curve has a long tail pointing to the right.
- 4 **Family of Distributions:** Shaped by **two separate degrees of freedom:** Numerator df_1 and Denominator df_2 .



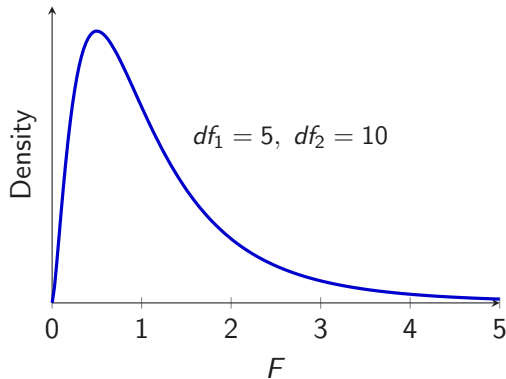
Visualizing the F -Distribution Curve

- Notice that the distribution starts at 0 and is heavily positively skewed.
- As degrees of freedom (df_1 and df_2) increase, the curve becomes less skewed and more bell-shaped.
- Rejection regions for variance tests and ANOVA are placed in the **right tail**.



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Reading the F -Distribution Table

Looking up critical values in Appendix B.6 requires identifying three parameters:

- ❶ **The Level of Significance (α)** (Appendix B.6A is for $\alpha = 0.05$, B.6B is for $\alpha = 0.01$).
- ❷ **Numerator Degrees of Freedom (df_1)**: Read across the top columns.
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Practice Lookup:

Find the critical F -value for $\alpha = 0.05$ with $df_1 = 6$ in the numerator and $df_2 = 7$ in the denominator.



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Practice Lookup:

Find the critical F -value for $\alpha = 0.05$ with $df_1 = 6$ in the numerator and $df_2 = 7$ in the denominator.

→ **Critical** $F_{6,7,0.05} = 3.87$



Pop-Up Question 1: Table Reading

Discussion

Pop-Up Question!

Use your statistical tables (Appendix B.6A) to find the critical F -values at $\alpha = 0.05$:

1. $df_1 = 4$ (numerator), $df_2 = 12$ (denominator).
2. $df_1 = 10$ (numerator), $df_2 = 20$ (denominator).



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Answers:

- 1: $F_{4,12,0.05} = 3.26$
- 2: $F_{10,20,0.05} = 2.35$



Testing Two Population Variances

Comparing Two Population Variances

Before we can run an ANOVA test (or to evaluate consistency between two suppliers), we test whether two independent population variances are equal:

$$H_0 : \sigma_1^2 = \sigma_2^2 \quad \text{vs.} \quad H_1 : \sigma_1^2 \neq \sigma_2^2$$



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The F -Test Statistic Formula:

$$F = \frac{s_1^2}{s_2^2}$$

Where s_1^2 is the sample variance of Group 1 and s_2^2 is the sample variance of Group 2.



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Where s_1^2 is the sample variance of Group 1 and s_2^2 is the sample variance of Group 2.

The Universal Convention: Place the **larger sample variance in the numerator** ($s_1^2 \geq s_2^2$). This forces $F \geq 1.0$, allowing us to always use the right tail of the F -distribution!

Example 1: Lammers Limos (Driving Consistency)

Textbook Example: Lammers Limousines

Lammers Limos offers airport transportation in Detroit. President Sean Lammers is evaluating two driving routes to the airport: **U.S. 25** and **Interstate 75**.

He collected sample driving times (in minutes):

- **U.S. 25** ($n_1 = 7$): 52, 67, 56, 45, 70, 54, 64 ($\bar{x}_1 = 58.29, s_1^2 = 80.90$)
- **I-75** ($n_2 = 8$): 59, 60, 61, 51, 56, 63, 57, 65 ($\bar{x}_2 = 59.00, s_2^2 = 19.14$)

Task: At the **0.10 significance level**, is there a difference in the **variation** of driving times between the two routes?



Lammers Limos: Steps 1 & 2

Step 1: State the Hypotheses.

- $H_0 : \sigma_1^2 = \sigma_2^2$ (Variances are equal)
- $H_1 : \sigma_1^2 \neq \sigma_2^2$ (Variances are different)



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Step 2: Select the Level of Significance.

- $\alpha = 0.10$.
- Because this is a two-tailed test, we divide α by 2 for the right-tail lookup: $\alpha/2 = \mathbf{0.05}$.



Lammers Limos: Steps 3 & 4

Step 3: Select the Test Statistic.

- Use the F -statistic: $F = s_1^2/s_2^2$.



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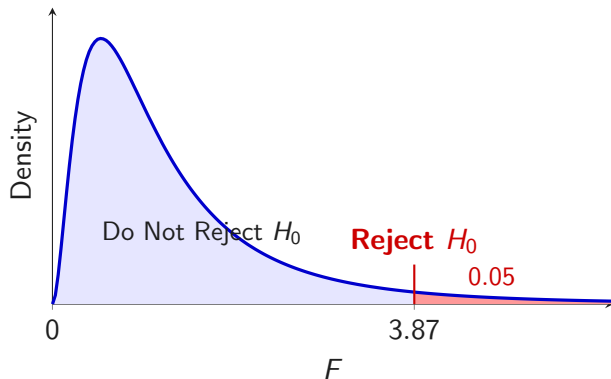
Step 4: Formulate the Decision Rule.

- Put the larger variance ($s_1^2 = 80.90$) in the numerator $\rightarrow df_1 = n_1 - 1 = 7 - 1 = 6$.
- Put the smaller variance ($s_2^2 = 19.14$) in the denominator $\rightarrow df_2 = n_2 - 1 = 8 - 1 = 7$.
- Look up $F_{6,7,0.05}$ in Appendix B.6A: **Critical Value = 3.87**.
- **Decision Rule:** Reject H_0 if computed $F > 3.87$.



Lammers Limos: Visualizing the Decision Rule

Rejection Region for $df_1 = 6$, $df_2 = 7$, $\alpha = 0.05$



Lammers Limos: Steps 5 & 6

Step 5: Compute the Test Statistic & Make Decision.

$$F = \frac{s_1^2}{s_2^2} = \frac{80.90}{19.14} = \mathbf{4.23}$$



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Because $4.23 > 3.87$, it falls into the rejection region. We **Reject** H_0 .



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Step 6: Interpret the Result.

- There is a statistically significant difference in the variability of travel times between the two routes.
- **Business Takeaway:** Even though both routes take about 58-59 minutes on average, Interstate 75 ($s^2 = 19.14$) is far more consistent and predictable than U.S. 25 ($s^2 = 80.90$). Lammers should schedule drivers on I-75!



Example 2: Google Page Speed Redesign

Textbook Exercise 16: Website Load Times

A women's clothing retailer redesigned its website to improve image loading times. A sample of **16 load times** for the **new page** had a standard deviation of $s_1 = 0.22$ seconds.

A sample of **16 load times** for the **current page** had a standard deviation of $s_2 = 0.12$ seconds.

Task: At the **0.05 significance level**, can we conclude that there is **more variation** in the load time of the new page?



1. Hypotheses (One-Tailed):

- $H_0 : \sigma_{\text{new}}^2 \leq \sigma_{\text{current}}^2$
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2. Critical Value:

- $df_1 = 16 - 1 = 15$.
- $df_2 = 16 - 1 = 15$.
- One-tailed test, $\alpha = 0.05$.
- Table B.6A $\rightarrow$ **Critical** $F_{15,15,0.05} = 2.40$.

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$$s_1^2 = (0.22)^2 = 0.0484$$

$$s_2^2 = (0.12)^2 = 0.0144$$

$$F = \frac{0.0484}{0.0144} = \mathbf{3.36}$$



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Decision: Since $3.36 > 2.40$, we **Reject** H_0 . The new web page has significantly higher variance in load times!



15 Minute Break

Introduction to One-Way ANOVA

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Three Assumptions Required for ANOVA:

- 1 The populations follow a **normal distribution**.
- 2 The populations have **equal standard deviations** ($\sigma_1 = \sigma_2 = \cdots = \sigma_k$).
- 3 The samples are **random and independent** from interval or ratio scale data.



Formulating Hypotheses for One-Way ANOVA

In One-Way ANOVA, the hypotheses are always structured as follows:

- **Null Hypothesis (H_0):** All population means are equal.

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- **Alternate Hypothesis (H_1):** Not all population means are equal.

H_1 : At least one population mean is different.

H_1 does **not** state that *all* means are different! It only requires that at least one treatment group differs from the others.



The Logic of ANOVA: Decomposing Variation

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1. Treatment Variation (SST):

- Measures differences **between** group sample means and the grand mean.

2. Error Variation (SSE):

- Measures random differences **within** each group.



The ANOVA Table Structure

All calculations are organized into a standardized **ANOVA Table**:

Source of Variation	Sum of Squares (SS)	Degrees of Freedom (df)	Mean Square (MS)
Treatments	SST	$k - 1$	$MST = \frac{SST}{k-1}$
Error	SSE	$n_T - k$	$MSE = \frac{SSE}{n_T - k}$
Total	SS_{total}	$n_T - 1$	

Where:

- k = Number of treatment groups.
- n_T = Total number of observations across all groups.



Equations for ANOVA Calculations

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4. F -Statistic:

$$F = \frac{MST}{MSE} = \frac{SST / (k - 1)}{SSE / (n_T - k)}$$



Example 3: Airline Passenger Satisfaction

Textbook Example: Airline Service Ratings

A travel research firm surveyed passengers across 3 major airlines (Northern, Wasp, and PAI) and asked them to rate their satisfaction on a scale from 1 to 10.

Sample Ratings:

- **Northern Airlines** ($n_1 = 4$): 6, 8, 4, 6 ($\bar{X}_1 = 6.0$)
- **Wasp Airlines** ($n_2 = 5$): 8, 9, 6, 7, 10 ($\bar{X}_2 = 8.0$)
- **PAI Airlines** ($n_3 = 6$): 3, 4, 2, 6, 4, 5 ($\bar{X}_3 = 4.0$)

Task: At the **0.05 significance level**, is there a difference in the mean satisfaction ratings among the three airlines? ($n_T = 15, k = 3$).



Airline Ratings: Step 1 - Find Grand Mean

First, calculate the **Grand Mean** ($\bar{X}_G$) across all 15 passengers:

$$\bar{X}_G = \frac{\sum X}{n_T} = \frac{(6 + 8 + 4 + 6) + (8 + 9 + 6 + 7 + 10) + (3 + 4 + 2 + 6 + 4 + 5)}{15}$$



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Group Means Summary:

- Northern: $\bar{X}_1 = 6.00$ ($n_1 = 4$)
- Wasp: $\bar{X}_2 = 8.00$ ($n_2 = 5$)
- PAI: $\bar{X}_3 = 4.00$ ($n_3 = 6$)



Airline Ratings: Step 2 - Calculate SST

Compute the **Treatment Sum of Squares** (SST):

$$SST = \sum n_c (\bar{X}_c - \bar{X}_G)^2$$



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$$\begin{aligned} SST &= 4(6.00 - 5.87)^2 + 5(8.00 - 5.87)^2 + 6(4.00 - 5.87)^2 \\ &= 4(0.13)^2 + 5(2.13)^2 + 6(-1.87)^2 \\ &= 4(0.0169) + 5(4.5369) + 6(3.4969) \\ &= 0.0676 + 22.6845 + 20.9814 = \mathbf{43.73} \end{aligned}$$



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Airline Ratings: Step 3 - Calculate SS_{total} & SSE

Summing squared deviations of all 15 observations from $\bar{X}_G = 5.87$ yields:

$$SS_{\text{total}} = 58.93$$



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Now subtract SST from SS_{total} to find SSE :

$$SSE = SS_{\text{total}} - SST = 58.93 - 43.73 = \mathbf{15.20}$$



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$$SSE = SS_{\text{total}} - SST = 58.93 - 43.73 = \mathbf{15.20}$$

Calculate Degrees of Freedom:

- Treatment $df_1 = k - 1 = 3 - 1 = \mathbf{2}$.
- Error $df_2 = n_T - k = 15 - 3 = \mathbf{12}$.
- Total $df = n_T - 1 = 15 - 1 = 14$.



Airline Ratings: Step 4 - Construct ANOVA Table

Now plug everything into the ANOVA table:

Source	<i>SS</i>	<i>df</i>	$MS = SS/df$	<i>F</i> -Ratio
Treatments	43.73	2	$MST = 43.73/2 = \mathbf{21.87}$	$F = \frac{21.87}{1.27} = \mathbf{17.22}$
Error	15.20	12	$MSE = 15.20/12 = \mathbf{1.27}$	
Total	58.93	14		



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Critical Value Lookup:

- For $df_1 = 2, df_2 = 12$ at $\alpha = 0.05$ (Table B.6A) $\rightarrow$ **Critical Value = 3.89.**



Airline Ratings: Decision & Interpretation

Mathematical Decision:

- Computed $F = 17.22$.
- Critical $F = 3.89$.



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Business Conclusion:

- There is a statistically significant difference in passenger satisfaction ratings among Northern, Wasp, and PAI airlines.
- Looking at the sample means, Wasp ($\bar{X} = 8.0$) scored noticeably higher than PAI ($\bar{X} = 4.0$).



Software Application: Reading Excel ANOVA Output

In business, software computes the ANOVA table instantly! Compare our manual calculations to this **Excel ANOVA Output** for the same problem:

ANOVA: Single Factor						
<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
Between Groups	43.733	2	21.867	17.218	0.0003	3.885
Within Groups	15.200	12	1.267			
Total	58.933	14				



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Total	58.933	14				

Software Decision Rule: Because the $P\text{-value} = 0.0003 < 0.05$, we reject H_0 . It is virtually impossible this difference happened by chance!



Pop-Up Question 2: Reading Software Output

Discussion

Pop-Up Question!

You run an ANOVA in Excel comparing 4 different retail displays ($k = 4$). The output shows:

Between Groups SS: 120.5, $df = 3$

Within Groups SS: 450.0, $df = 36$

P-value: 0.082

1. What is MSE ?
2. At $\alpha = 0.05$, do we reject H_0 ?



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P-value: 0.082

1. What is MSE ?
2. At $\alpha = 0.05$, do we reject H_0 ?

Answers:

- 1: $MSE = \text{Within SS} / df = 450.0 / 36 = 12.5$.
- 2: Since $P\text{-value}(0.082) > 0.05$, we **Do Not Reject** H_0 . There is insufficient evidence to

