Lecture 02

Fundamentals of Business Mathematics

MATH-114 • BBA 2k26

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Lecture 02 Outline: Linear Equations & Applications

Part 1: Linear Equations in Two & n Variables

- Algebraic Structure & Solution Sets
- Continuous vs. Discrete Variables in Business
- Production Possibilities Modeling

Part 2: Graphical Representation & Intercepts

- Coordinate Axes & Intercept Method
- Equations Passing Through the Origin
- Horizontal ($y = k$) & Vertical ($x = k$) Lines

Part 3: Rate of Change & Slope Dynamics

- Mathematical Definition of Slope (m)
- Positive, Negative, Zero, & Undefined Slopes

Part 4: Slope-Intercept Form & Business Models

- Slope-Intercept Form ($y = mx + k$)
- Base Salary & Commission Modeling
- Vehicle Fleet Operating Cost Model
- Fixed vs. Variable Cost Pivots & Shifts

Part 5: Formulating Equations & Line Relations

- Point-Slope & Two-Point Methods
- Parallel ($m_1 = m_2$) & Perpendicular Lines
- Nuclear Power Growth Forecasting
- Straight-Line Machine Depreciation

Linear Equations in Two & n Variables

Algebraic Definition of Linear Equations (Section 2.1)

Linear Equation in n Variables (Budnick, p. 34)
A linear equation in n variables $x_1, x_2, \dots, x_n$ is an equation that can be written in the form:

$$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n = b$$

where $a_1, a_2, \dots, a_n$ and b are real-valued constants, and not all $a_i = 0$.

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- **Commercial Relevance**: Represents constant marginal return/cost systems.

Solution Sets & Domain Constraints

Solution Sets for Two-Variable Linear Equations

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- *Examples*: Number of cars, laptops, employees.
- **Non-negativity**: $x \geq 0, y \geq 0$ restricts economic problems to Quadrant I.

Example 1: Solution Set Determination

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Step-by-Step Algebraic Solution:

Part 1: Given $x = -2$

$$\begin{aligned} 2(-2) + 4y &= 16 \\ -4 + 4y &= 16 \\ 4y &= 20 \implies \mathbf{y = 5} \end{aligned}$$

Ordered pair solution: $(-\mathbf{2}, \mathbf{5})$.

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Part 2: Given $y = 0$

$$\begin{aligned} 2x + 4(0) &= 16 \\ 2x &= 16 \implies \mathbf{x = 8} \end{aligned}$$

Ordered pair solution: $(\mathbf{8}, \mathbf{0})$.

Example 2: Production Possibilities Model

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- **Scenario A**: If management decides to produce $y = 30$ units of Product B:

$$3x + 2.5(30) = 120 \implies 3x + 75 = 120 \implies 3x = 45 \implies \mathbf{x} = \mathbf{15} \text{ units of Product A}$$

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- **Scenario B (Max Product A)**: Set $y = 0 \implies 3x = 120 \implies \mathbf{x} = \mathbf{40}$ units.
- **Scenario C (Max Product B)**: Set $x = 0 \implies 2.5y = 120 \implies \mathbf{y} = \mathbf{48}$ units.

In-Class Activity: Production Trade-offs

Discussion

Marginal Trade-offs

Consider the production equation $3x + 2.5y = 120$:

1. How many units of Product B must be sacrificed to produce 1 additional unit of Product A?

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1. How many units of Product B must be sacrificed to produce 1 additional unit of Product A?
2. What is the mathematical trade-off ratio $(\frac{\Delta y}{\Delta x})$?
3. If total available labor hours expand from 120 to 150 hours, how does the production capacity line shift?

Graphical Representation & Intercepts

Geometry of Two-Variable Equations (Section 2.2)

Axiom of Two-Point Linearity

The graph of any two-variable linear equation $ax + by = c$ is a straight line. Since two points uniquely define a line, we only need to identify **two distinct points** to graph the entire solution set.

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   - Point where the line crosses the horizontal axis.
   - Represents break-even or threshold input.

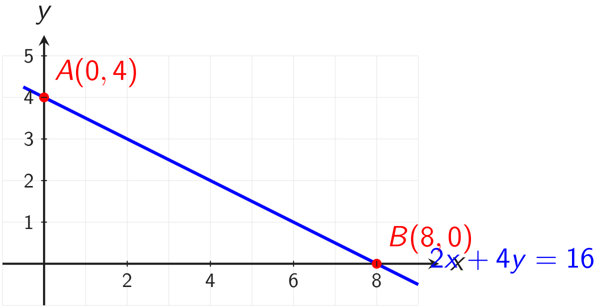
Example 5: Graphing via Intercepts

Equation: $2x + 4y = 16$

Find y-intercept ($x = 0$):

$$2(0) + 4y = 16 \implies 4y = 16 \implies \mathbf{y = 4}$$

Point A: $(\mathbf{0}, \mathbf{4})$.



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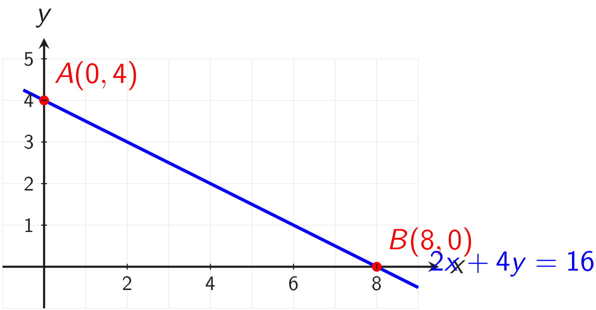

Equations Passing Through the Origin

Zero Constant Term ($c = 0$): Equations of the form $ax + by = 0$ pass directly through the origin $(0, 0)$. For these equations, both the x-intercept and y-intercept coincide at $(0, 0)$. A second non-zero point must be picked!

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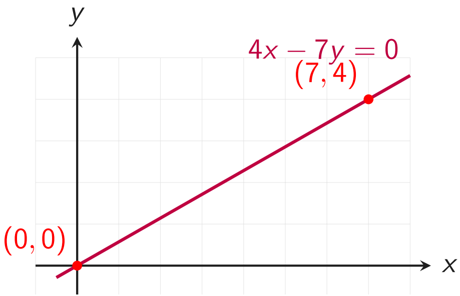
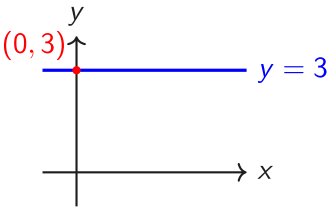


Special Cases: Horizontal & Vertical Lines

Horizontal Lines: $y = k$

- y remains constant at k for all values of x.



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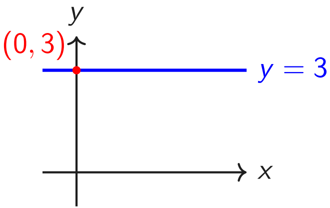

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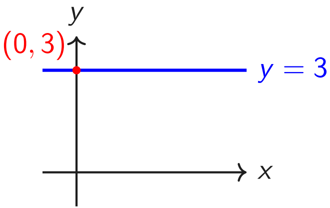

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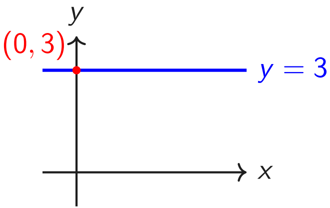

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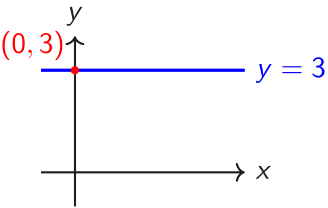

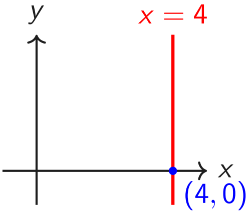

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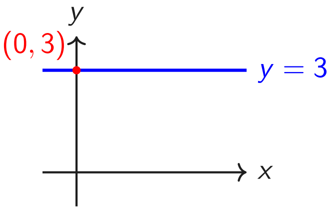

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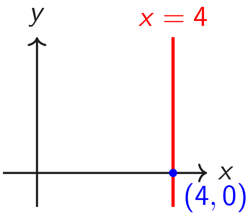

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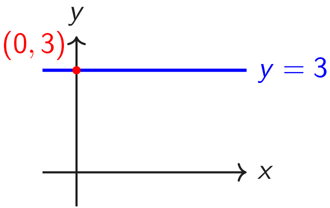

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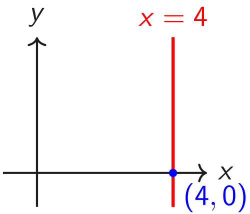

Rate of Change & The Concept of Slope

Defining Slope as Rate of Change (Section 2.2)

Mathematical Definition of Slope

The slope m of a non-vertical line passing through two points (x_1, y_1) and (x_2, y_2) is defined as the ratio of the vertical change (Δy) to the horizontal change (Δx):

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{where } x_1 \neq x_2$$

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$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{where } x_1 \neq x_2$$

Economic Meaning of Slope:

Slope represents the **rate of change** of y per 1-unit increase in x. In business economics, slope measures marginal values:

- Marginal Cost $(\frac{\Delta C}{\Delta q})$, Marginal Revenue $(\frac{\Delta R}{\Delta q})$, Marginal Tax Rate $(\frac{\Delta T}{\Delta I})$.

Four Directional Behaviors of Slope

Slope	Graph Direction	Economic Relationship	Business Example
$\mathbf{m > 0}$	Rises left to right	Direct / Positive	Total Cost vs. Volume
$\mathbf{m < 0}$	Falls left to right	Inverse / Negative	Price vs. Quantity Demanded
$\mathbf{m = 0}$	Horizontal line	Constant y regardless of x	Fixed Lease Rent
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Sign Convention Warning: Pay strict attention to signs! A negative slope ($m < 0$) indicates that as x increases, y decreases at a constant rate.

Example 7: Slope Calculation & Geometry

Problem: Calculate slope connecting $(2, 4)$ and $(5, 12)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 4}{5 - 2} = \mathbf{\frac{8}{3}} \approx \mathbf{2.67}$$

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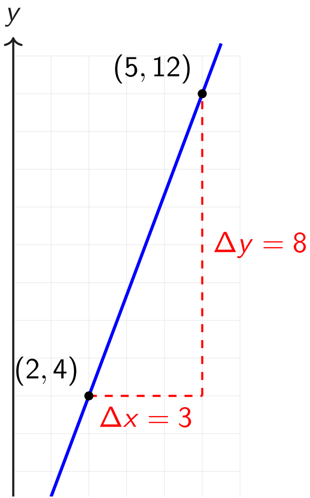

Example 8: Undefined Slope of Vertical Lines

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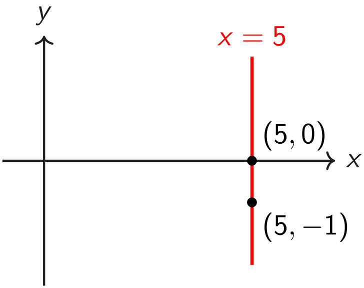

Slope-Intercept Form & Commercial Interpretation

Slope-Intercept Form Transformation (Section 2.3)

Slope-Intercept Form

$$y = mx + k$$

where m is the slope of the line, and k is the y-intercept $(0, k)$.

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Commercial Application 1: Commission Salary Model

Sales commission salary equation:

$$y = 3x + 25$$

where $x =$ units sold per week, $y =$ total weekly salary (\$).

- **Base Salary ($k = 25$)**: Fixed weekly base pay of **\$25** even if zero units are sold ($x = 0$).

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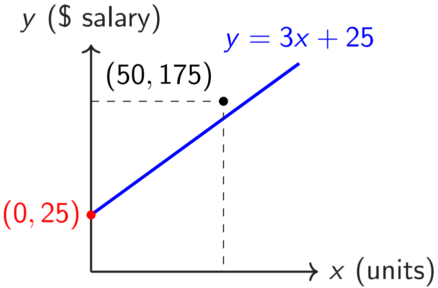

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- **Application**: Total annual cost for a vehicle driven **25,000 miles**:

$$C = 0.40(25,000) + 18,000 = 10,000 + 18,000 = \mathbf{\$28,000}$$

Cost Model Shifts vs. Pivots

Discussion

Graphing Cost Changes: Given $C = 0.40x + 18,000$

1. If annual vehicle insurance premiums increase by \$2,000 per vehicle, how does the equation change? Does the graph **shift** or **pivot**?

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Formulating Equations of Straight Lines

Three Cases for Finding Equations of Lines (Section 2.4)

Case	Given Information	Primary Formula / Approach
Case 1	Slope m and y-intercept $(0, k)$	Direct Slope-Intercept: $y = mx + k$
Case 2	Slope m and one Point (x_1, y_1)	Point-Slope Form: $y - y_1 = m(x - x_1)$
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Point-Slope Formula

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Case 1 & Case 2 Examples

Example 13 (Case 1):

- Given: Slope $m = -5$, y-intercept $(0, 15)$.

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$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 10 &= 5(x - (-4)) \\ y - 10 &= 5(x + 4) \\ y - 10 &= 5x + 20 \\ \mathbf{y} &= \mathbf{5x + 30} \end{aligned}$$

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Example 17 Problem: Given line $2x - 6y = 24$.
Find the perpendicular line passing through $(2, 5)$.

1. Slope of given line: $6y = 2x - 24 \implies y = \frac{1}{3}x - 4 \implies m_1 = \frac{1}{3}$.
2. Perpendicular slope: $m_2 = -3$.
3. Apply Point-Slope at $(2, 5)$:

$$y - 5 = -3(x - 2) \implies y - 5 = -3x + 6$$

$$\mathbf{3x + y = 11} \quad \text{or} \quad \mathbf{y = -3x + 11}$$

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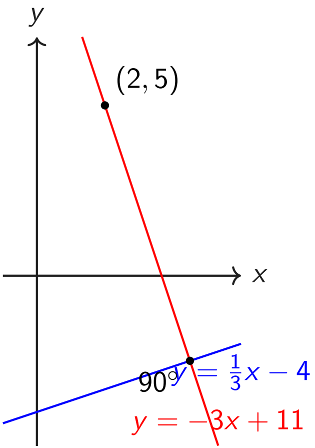

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Step 3: Forecast for Year 2000 ($t = 20$)

$$p(20) = 0.0096(20) + 0.1144 = 0.192 + 0.1144 = \mathbf{0.3064} \implies \mathbf{30.64\%}$$

Straight-Line Asset Depreciation Model

Business Context: Industrial machine purchased for **$18,000** ($t = 0, V = \$18,000$). After 1 year of operation ($t = 1$), book value drops to **$14,500** ($t = 1, V = \$14,500$).

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  - **Useful Life (Scrap Value $0)**: $3,500t = 18,000 \implies \mathbf{t \approx 5.14}$ years.

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Lecture 02 Summary

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Homework Problems (Budnick, 4th Ed)

- **Section 2.1**: Ex 1, 5, 9, 15, 20 (p. 41)
- **Section 2.2**: Ex 3, 9, 21, 31, 38, 42 (pp. 51–52)
- **Section 2.3**: Ex 1, 5, 11, 25, 29 (pp. 55–56)
- **Section 2.4**: Ex 3, 11, 21, 31, 37 (pp. 63–64)

Practice these textbook problems to prepare for **Assignment and Quiz 1**!