



Lecture 05

Fundamentals of Business Mathematics

MATH-114 • BBA 2k26

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Lecture 05 Outline: Mathematical Functions

Part 1: Concept & Definition of Functions

- Functions as Mathematical Rules, Mapping Notation ($f : x \rightarrow y$) & Relations, Independent vs. Dependent Variables

Part 2: Evaluating Functions & Business Models

- Functional Evaluation Notation $f(b)$, Salesperson Weekly Commission Model, Police Fleet Vehicle Cost Function

Part 3: Mathematical Domain & Range Analysis

- Pure Algebraic Domain Restrictions, Polynomials, Quotients, & Radical Functions, Solving Quadratic Radical Inequalities

Part 4: Restricted Business Domains & Piecewise Rules

- Economic Non-Negativity & Capacity Limits, Integer Production Constraints, Piecewise Commercial Driver Wage Functions

Part 5: Multivariate Functions & Subscripted Models

- Bivariate Functions $z = f(x, y)$ & Ordered Pairs, Domain Constraints in 3D Space, Subscripted Notation $y = f(x_1, \dots, x_n)$, Applied Case: Military Defense Procurement



Concept & Definition of Functions

Functions as Input-Output Transformations

Definition: Mathematical Function (Budnick, Section 4.1)

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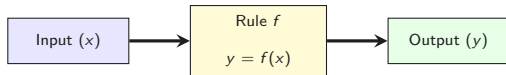
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Algebraic Example: $y = x^2 - 2x + 1$

- Input $x = 1 \implies y = 1^2 - 2(1) + 1 = 0$.
- Input $x = -5 \implies y = (-5)^2 - 2(-5) + 1 = 36$.
- Input $x = 10 \implies y = 10^2 - 2(10) + 1 = 81$.



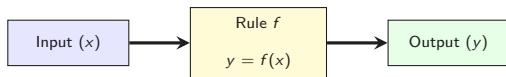
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Domain & Range Definitions:

- **Domain (D)**: Set of all possible *input* values (x).
- **Range (R)**: Set of all resulting *output* values (y).

Mapping Notation & Valid Function Rules

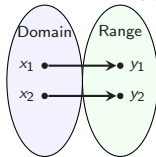
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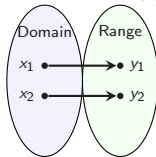
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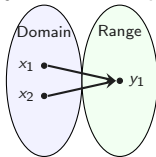
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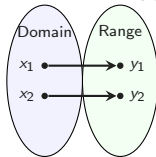
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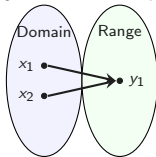
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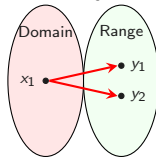
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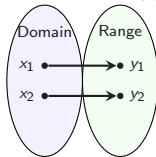
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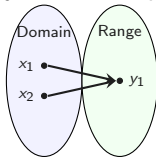
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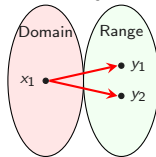
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Variable Classifications:

- **Independent Variable (x):** The input variable whose values can be freely assigned.
- **Dependent Variable (y):** The output variable whose value depends entirely on x .



Evaluating Functions & Business Models

Functional Evaluation: Salesperson Commission Model

Functional Notation $f(b)$: For any function $y = f(x)$, the output corresponding to input $x = b$ is denoted as $f(b)$, evaluated by substituting b for every occurrence of x .



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 $y = f(x) = 3x + 25$ where x is units sold/week and y is weekly salary (\$).



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Evaluating Specific Sales Levels:

- **Sell 100 units:**
 $f(100) = 3(100) + 25 = \$325$.
- **Sell 75 units:** $f(75) = 3(75) + 25 = \$250$.
- **Zero sales ($x = 0$):**
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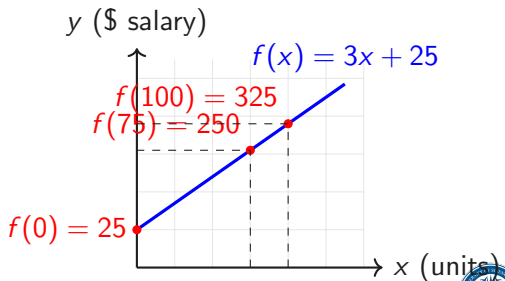
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Problem Statement: Given the quadratic functional relationship:

$$z = h(t) = t^2 + t - 10$$

Evaluate the function at: $t = 0$, -5 , and $h(u + v)$.



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Step-by-Step Algebraic Evaluation:

- **Part (a):** Substitute $t = 0$:

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- **Part (c):** Substitute expression $t = (u + v)$ wherever t appears:

$$h(u + v) = (u + v)^2 + (u + v) - 10 = \mathbf{u^2 + 2uv + v^2 + u + v - 10}$$



Commercial Application: Police Fleet Vehicle Cost Model

Fleet Cost Structure: A city police department constructs a total cost function $C = f(x)$ for patrol vehicles as a function of total miles driven (x):

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Cost Component Analysis:

- **Fixed Acquisition Cost:**
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Evaluating Mileage Projections:

- **Driven 50,000 miles:**

$$C(50,000) = 0.40(50,000) + 18,000 = \$38,000$$

- **Driven 100,000 miles:**

$$C(100,000) = 0.40(100,000) + 18,000 = \$58,000$$

Mathematical Domain & Range Analysis

Pure Algebraic Domain Rules

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Quotients $\frac{g(x)}{h(x)}$ are undefined when the denominator equals zero:

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2. Even Roots of Negatives

Radical expressions $\sqrt{g(x)}$ yield imaginary numbers if the radicand is negative:

$$g(x) \geq 0$$

Restrict x such that $g(x)$ is non-negative.

Domain Analysis: Polynomials & Rational Functions

Polynomial Functions

$$y = f(x) = x^2 - 2x + 1$$

- Contains no denominators or square roots.
- Any real number substituted for x produces a unique real output y .
- **Domain (D):**

$$D = \{x \mid x \text{ is real}\} = (-\infty, \infty)$$



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Rational Functions

$$u = f(v) = \frac{1}{v^2 - 4}$$

- Exclude values where denominator $v^2 - 4 = 0$.
- Factor: $(v - 2)(v + 2) = 0 \implies v = \pm 2$.
- **Domain (D):**

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Practice Exercise:

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Discussion

Practice Exercise:

Find the domain of $y = f(x) = \sqrt{10 - x}$.

Solution:

$$10 - x \geq 0 \implies x \leq 10.$$

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- **Sign Analysis for Product** $(x + 4)(x - 3) \geq 0$:
 - ▶ *Both Factors Positive:* $x + 4 > 0$ and $x - 3 > 0 \implies x \geq 3$.
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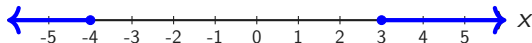


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Domain Result: $D = \{x \mid x \leq -4 \text{ or } x \geq 3\} = (-\infty, -4] \cup [3, \infty)$.



Restricted Business Domains & Piecewise Rules

Restricted Domain & Range in Business Contexts

Practical Business Restrictions: While a function may mathematically accept all real numbers, operational conditions (physical capacity, non-negativity, policy limits) impose a **restricted domain** and corresponding **restricted range**.



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2. Discrete Integer Constraints

$$y = 3x + 25$$

- If x represents physical units sold (e.g., laptops, cars), fractional inputs make no economic sense.
- **Restricted Domain:**

$$D = \{x \mid x \text{ is an integer and } 0 \leq x \leq u\}$$



Piecewise-Defined Commercial Functions

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Application: Long-Distance Driver Wage Contract

A trucking firm pays drivers based on trip duration n (hours):

$$w = f(n) = \begin{cases} 30.5 - 1.5n, & 0 \leq n < 15 \\ 8.00, & n \geq 15 \end{cases}$$

where w = hourly wage (\$) and n = trip duration (hours).



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- **Short Trip** ($n = 10$ hrs):

$$w = f(10) = 30.5 - 1.5(10) = \text{\$15.50/hr}$$

- **Long Trip** ($n = 18$ hrs):

$$w = f(18) = \text{\$8.00/hr} \quad (n \geq 15)$$



Multivariate Functions & Subscripted Models

Multivariate & Bivariate Functions

Multivariate Functions: Relationships where a dependent variable depends on **more than one** independent variable ($z = f(x, y)$ or $y = f(x_1, x_2, \dots, x_n)$). Most realistic corporate models are multivariate (e.g., profit depends on price, advertising, raw material costs, and labor).



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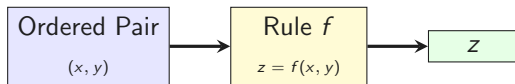


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Domain Constraints for Bivariate Functions

Bivariate Quotient Function:

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Domain Exclusion Condition:

- Numerator and denominator are individually defined for all real (x, y) .
- Function is undefined where denominator $3x - y = 0 \implies y = 3x$.
- **Domain (D):**

$$D = \{(x, y) \mid 3x - y \neq 0\}$$



Domain Constraints for Bivariate Functions

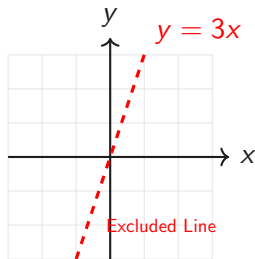
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Subscripted Variables for n -Variable Systems

Subscripted Notation: When a model involves $n > 3$ independent variables, using different alphabet letters becomes cumbersome. We use subscripted variables $x_1, x_2, \dots, x_n$:

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Evaluation: Given a 4-variable function:

$$y = f(x_1, x_2, x_3, x_4) = x_1^2 - 2x_1x_2 + x_3^2x_4 - 25$$



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$$y = f(x_1, x_2, x_3, x_4) = x_1^2 - 2x_1x_2 + x_3^2x_4 - 25$$

Evaluate $f(-2, 0, 1, 4)$ where $x_1 = -2, x_2 = 0, x_3 = 1, x_4 = 4$:

$$\begin{aligned} f(-2, 0, 1, 4) &= (-2)^2 - 2(-2)(0) + (1)^2(4) - 25 \\ &= 4 - 0 + 4 - 25 = -17 \end{aligned}$$



Applied Case Study: Defense Procurement Contract

Contract Terms: US Defense Dept contracts a manufacturer for two gas mask models priced at \$175 (x_1) and \$225 (x_2). If combined weekly volume exceeds 5,000 units ($x_1 + x_2 > 5,000$), a bonus of \$50,000 plus \$25/unit for all units over 5,000 is paid.



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Evaluating Weekly Revenue for $x_1 = 3,000$ and $x_2 = 3,000$ ($x_1 + x_2 = 6,000 > 5,000$):

$$f(3000, 3000) = 200(3000) + 250(3000) - 75,000 = 600,000 + 750,000 - 75,000 = \$1,275,000$$



Discussion: Contract Function Simplification

Discussion

Point for Thought & Discussion

Examine the sales function for $x_1 + x_2 > 5,000$:

$$f(x_1, x_2) = 200x_1 + 250x_2 - 75,000$$

- ① Why does it appear that **all** units get the \$25 bonus ($175 \rightarrow 200$ and $225 \rightarrow 250$)?
- ② Where is the \$50,000 fixed bonus in this simplified function?
- ③ What does the constant term $-\$75,000$ represent algebraically?



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Algebraic Proof of Equivalence:

$$\begin{aligned}\text{Revenue} &= 175x_1 + 225x_2 + 50,000 + 25(x_1 + x_2 - 5,000) \\ &= 175x_1 + 225x_2 + 50,000 + 25x_1 + 25x_2 - 125,000 = \mathbf{200x_1 + 250x_2 - 75,000} \quad \checkmark\end{aligned}$$



Summary & Homework Assignment

Summary & Homework Practice Assignment

Lecture 05 Summary

- A function maps each input x to exactly one output y .
- Algebraic domain restrictions require $h(x) \neq 0$ and $g(x) \geq 0$ for $\sqrt{g(x)}$.
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Homework Exercises (Budnick, 4th Ed, Sec 4.1)

- **Evaluation:** Exercises 1, 5, 9, 15
- **Domain Analysis:** Exercises 17, 21, 27, 31, 38, 40
- **Applied Business Models:**
 - ▶ Ex 41 (Cost Function Domain/Range)
 - ▶ Ex 44, 45 (Term Life Insurance Premiums)
 - ▶ Ex 48 (Car Rental Rates)
 - ▶ Ex 51 (Membership Incentive Discount)
 - ▶ Ex 63, 64 (Multivariate Commission Salary)

