



Lecture 06

Fundamentals of Business Mathematics

MATH-114 • BBA 2k26

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Taxonomy & Types of Functions

Constant & Linear Functions

Functions can be classified by their algebraic structure. Recognizing the type of function allows us to anticipate its behavior and graph.

1. Constant Functions: $f(x) = a_0$

- Domain is all real numbers; Range is a single value $\{a_0\}$.
- **Example:** Marginal Revenue in competitive markets, $MR = f(x) = 7.50$.

2. Linear Functions: $f(x) = a_1x + a_0$

- Graphs as a straight line with slope a_1 and y-intercept $(0, a_0)$.
- **Example:** Total Cost Model, $C(x) = 0.40x + 18,000$.



Quadratic & Polynomial Functions

3. Quadratic Functions: $f(x) = a_2x^2 + a_1x + a_0$ ($a_2 \neq 0$)

- Graphs as a parabola (concave up or concave down).
- **Example: Quadratic Demand Function**

$$q_d = f(p) = p^2 - 70p + 1,225$$

Evaluate demand at $p = \$10$ and $p = \$30$:

$$f(10) = (10)^2 - 70(10) + 1,225 = 100 - 700 + 1,225 = 625 \text{ units.}$$

$$f(30) = (30)^2 - 70(30) + 1,225 = 900 - 2,100 + 1,225 = 25 \text{ units.}$$

4. Polynomial Functions: $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$

- Exponents must be non-negative integers.
- The degree is the highest power n . Constant, linear, and quadratic are polynomials of degree 0, 1, and 2.



Rational Functions

5. Rational Functions: $f(x) = \frac{g(x)}{h(x)}$

- Formed by the ratio of two polynomial functions.
- Domain excludes any x where denominator $h(x) = 0$.

Example: Disability Rehabilitation Cost Model

Cost C (in thousands of \$) based on percentage of functionality recovered, x :

$$C = f(x) = \frac{5x}{120 - x} \quad \text{for } 0 \leq x \leq 100$$

Cost for a 30% recovery ($x = 30$):

$$f(30) = \frac{5(30)}{120-30} = \frac{150}{90} \approx 1.667 \text{ (or \$1,667)}$$



Combinations & Composite Functions

Algebraic Combinations

Just like numbers, functions can be combined using standard algebraic operations to form new functions:

- **Sum:** $p(x) = f(x) + g(x)$
- **Difference:** $q(x) = f(x) - g(x)$
- **Product:** $r(x) = f(x) \cdot g(x)$
- **Quotient:** $s(x) = \frac{f(x)}{g(x)}$ (provided $g(x) \neq 0$)

The domain of the combined function is the intersection of the domains of $f(x)$ and $g(x)$.



Composite Functions

A **composite function** exists when the output of an inner function becomes the direct input for an outer function.

Notation: $y = f(x) = g(h(x))$

Example: Let $y = g(u) = u^2 - 2u + 10$ and let $u = h(x) = x + 1$.

Find the composite function $f(x) = g(h(x))$:

$$\begin{aligned}f(x) &= g(x + 1) \\&= (x + 1)^2 - 2(x + 1) + 10 \\&= (x^2 + 2x + 1) - 2x - 2 + 10 \\&= x^2 + 9\end{aligned}$$



Evaluating Composite Functions

Example: Given $y = g(u) = 2u^3$ and $u = h(x) = x^2 - 2x + 5$.

Find the value of $g(h(2))$:

Step 1: Evaluate the inner function first.

$$h(2) = (2)^2 - 2(2) + 5 = 4 - 4 + 5 = 5$$

Step 2: Use the result as the input for the outer function.

$$g(5) = 2(5)^3 = 2(125) = 250$$



Composite Functions in Business Models

Salesperson Salary-Price Model: Linking sequential dependencies into a single direct model.

- **Salary** based on units sold: $y = g(x) = 2x + 50$
- **Demand** based on price: $x = h(p) = 150 - 2.5p$

To find salary directly from price, form $y = f(p) = g(h(p))$:

$$\begin{aligned}y &= 2(150 - 2.5p) + 50 \\&= 300 - 5p + 50 \\y &= 350 - 5p\end{aligned}$$

At a price of $p = \$30$, the expected salary is:

$$y = 350 - 5(30) = \$200.$$



Graphical Representation in 2-Space

Plotting & Scaling in 2-Space

- **Independent Variable:** Plotted on the horizontal axis (x -axis).
- **Dependent Variable:** Plotted on the vertical axis (y -axis).
- **Scaling:** Horizontal and vertical axes do not require identical intervals or units. Choose appropriate scales based on the domain and range of your business data.

Discussion

Why might the horizontal axis representing “Time (Years)” and the vertical axis representing “Corporate Revenue (\$ Millions)” have vastly different scales?

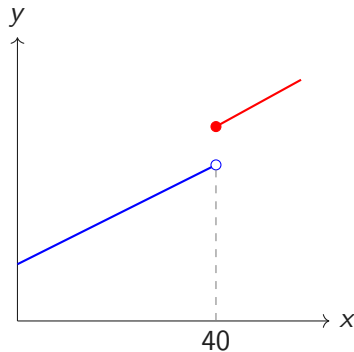


Graphing Piecewise Functions

Example: Piecewise Linear Model

$$y = f(x) = \begin{cases} 2x + 50, & 0 \leq x < 40 \\ 2.25x + 75, & x \geq 40 \end{cases}$$

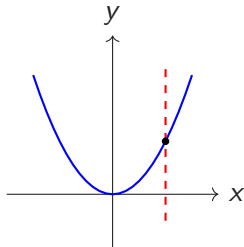
- If sales (x) are < 40 , base is \$50, commission \$2/unit.
- If sales (x) are ≥ 40 , base jumps to \$75, commission \$2.25/unit.
- Note the **open circle** at $(40, 130)$ and **closed circle** at $(40, 165)$.



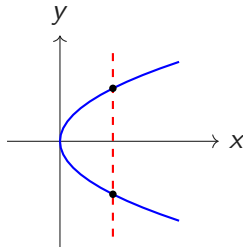
The Vertical Line Test

A relation is a function *if and only if* **every vertical line** drawn through its domain intersects the graph at **at most one point**.

Function



Not a Function



Homework

Homework Assignment

To prepare for the upcoming assessments, complete the following practice exercises from Chapter 4:

- **Section 4.2:**

- ▶ Function Classification: Exercises 1–24 (Odds)
- ▶ Applied Models: Exercises 29, 31, 35
- ▶ Composite Functions: Exercises 40–44

- **Section 4.3:**

- ▶ Graphing Practice: Exercises 1, 3, 11
- ▶ Vertical Line Test: Exercises 17 & 18

