



Lecture 07

Fundamentals of Business Mathematics

MATH-114 • BBA 2k26

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Linear Cost, Revenue, and Profit Functions

Linear Economic Functions & Base Assumptions

Linear Economic Models

Linear functions serve as baseline approximations in business. A single-variable economic function takes the form $y = f(x) = a_1x + a_0$, where the rate of change remains strictly constant.



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- **No Economies of Scale:** Ignores bulk purchasing discounts or capacity bottlenecks (overtime labor, machine wear).
- **Non-Negativity Constraints:** In real-world commercial scenarios, output, prices, and revenue cannot be negative ($x \geq 0, y \geq 0$).



Linear Cost Functions

Total Cost Equation:

$$C(x) = \text{Total Variable Cost} + \text{Total Fixed Cost} = v \cdot x + FC$$

where v is the variable cost per unit (slope) and FC is the fixed cost (y -intercept).



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- Vary directly with production volume x .
- Direct raw materials, assembly labor, packaging, power.
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Fixed Costs (FC)

- Incurred regardless of production level ($x = 0$).
- Factory lease, executive salaries, equipment insurance, depreciation.
- Represents y -intercept ($0, FC$).



Example: Multi-Department Linear Cost Model

Manufacturing Cost Scenario (Example 2):

A manufacturing firm incurs annual fixed costs of \$50,000. Variable costs include raw materials at \$5.50 per unit and labor across three assembly departments: Dept 1 (\$1.50), Dept 2 (\$0.75), and Dept 3 (\$1.25).



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Evaluation: At a volume of $x = 10,000$ units, total annual cost is:

$$C(10,000) = 9(10,000) + 50,000 = 90,000 + 50,000 = \textbf{\$140,000}$$



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Linear Revenue Functions

Total Revenue Function:

$$R(x) = (\text{Price per Unit}) \times (\text{Quantity Sold}) = p \cdot x$$

For n distinct products sold at prices $p_1, p_2, \dots, p_n$:

$$R(x_1, x_2, \dots, x_n) = p_1x_1 + p_2x_2 + \dots + p_nx_n = \sum_{j=1}^n p_jx_j$$



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- Flat rental rate: \$9.95 per car-day rented (d).



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Car Rental Agency Revenue (Example 3):

- Flat rental rate: \$9.95 per car-day rented (d).
- **Revenue Equation:** $R(d) = 9.95d$
- **Property:** Revenue passes through the origin $(0, 0)$ because zero rentals generate \$0 revenue.



Linear Profit Functions

Net Profit Function:

$$P(x) = R(x) - C(x) = p \cdot x - (v \cdot x + FC) = (\mathbf{p - v})\mathbf{x - FC}$$



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Marginal Profit / Contribution Margin ($p - v$):

The term $(p - v)$ represents the profit contribution per unit sold toward covering fixed costs (FC).



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- $P(x) > 0 \implies$ Operating Profit (Revenue exceeds Total Cost).



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- $P(x) > 0 \implies$ Operating Profit (Revenue exceeds Total Cost).
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- $P(x) < 0 \implies$ Operating Loss (Revenue fails to cover Total Cost).



Example: Profit Model & Target Evaluation

Commercial Equipment Business (Example 4):

Selling price $p = \$65$ per unit. Variable materials and labor costs are \$20.00 and \$27.50 per unit. Annual fixed costs are $FC = \$100,000$.



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$$P(x) = (65.00 - 47.50)x - 100,000 = \textbf{17.50x - 100,000}$$



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Step 3: Evaluate Profit at Volume $x = 20,000$ units

$$P(20,000) = 17.50(20,000) - 100,000 = 350,000 - 100,000 = \textbf{\$250,000}$$



Multivariate Profit Model: Multi-Farm Enterprise

Agricultural Conglomerate Model (Example 5):

An agribusiness operates three farms producing soybeans (x_1), corn (x_2), and potatoes (x_3). Total corporate administrative overhead is \$525,000.



Multivariate Profit Model: Multi-Farm Enterprise

Agricultural Conglomerate Model (Example 5):

An agribusiness operates three farms producing soybeans (x_1), corn (x_2), and potatoes (x_3). Total corporate administrative overhead is \$525,000.

Crop / Farm	Selling Price / Ton (p_j)	Variable Cost / Ton (v_j)	Unit Profit Contribution ($p_j - v_j$)
Soybeans (x_1)	\$900	\$500	\$400
Corn (x_2)	\$750	\$200	\$550
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Multivariate Profit Function:

$$P(x_1, x_2, x_3) = 400x_1 + 550x_2 + 450x_3 - 525,000$$



Other Examples of Linear Applications

Straight-Line Asset Depreciation Models

Straight-Line Depreciation assumes that an asset loses value at a constant rate over its useful life n .



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Case 1: Zero Salvage Value

$$V(t) = V_0 - d \cdot t$$

where V_0 = initial cost, $d = \frac{V_0}{n}$ = annual depreciation.



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Case 2: With Salvage Value (S_0)

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Commercial Significance: Book Value $V(t)$ tracks asset balance sheet value and provides annual tax depreciation deductions d .



Depreciation Comparison

Commercial Truck Acquisition ($V_0 = \$20,000$, Useful Life $n = 5$ years):



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Commercial Truck Acquisition ($V_0 = \$20,000$, **Useful Life** $n = 5$ **years**):

Scenario A: Zero Salvage Value

Scenario B: Salvage Value $S_0 = \$2,500$

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Scenario B: Salvage Value $S_0 = \$2,500$

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- $V(5) = \$2,500$ (Trade-in value)



Linear Demand Functions & Consumer Response

Linear Demand Equation:

$$q_d = f(p) = a_0 - a_1 p \quad (a_1 > 0)$$

where p is unit price, q_d is quantity demanded, $-a_1$ is the negative slope (Law of Demand), and a_0 is the q_d -intercept when $p = 0$.



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Example 7 (Deriving Demand Equation):

Market research identifies two price-quantity data points: $(p_1 = \$1, q_1 = 40,000)$ and $(p_2 = \$3, q_2 = 25,000)$.



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$$m = \frac{25,000 - 40,000}{3 - 1} = \frac{-15,000}{2} = -7,500$$



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Linear Supply Functions & Market Thresholds

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$$q_s = f(p) = a_1 p - a_0 \quad (a_1 > 0, a_0 > 0)$$

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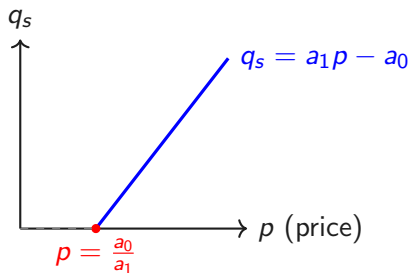
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Multi-Product Market Equilibrium Analysis

Two Interrelated Products Scenario (Example 9):

Market demand and supply equations for competing Products 1 and 2:

$$\begin{cases} q_{d1} = 100 - 2p_1 + 3p_2, & q_{s1} = 2p_1 - 4 \\ q_{d2} = 150 + 4p_1 - p_2, & q_{s2} = 3p_2 - 6 \end{cases}$$



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$$100 - 2p_1 + 3p_2 = 2p_1 - 4 \implies 4p_1 - 3p_2 = 104 \quad \text{— (Eq 1)}$$



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$$150 + 4p_1 - p_2 = 3p_2 - 6 \implies -4p_1 + 4p_2 = 156 \quad \text{— (Eq 2)}$$



Solving Multi-Product Equilibrium

Step 3: Solve the 2×2 Linear System

$$\begin{cases} 4p_1 - 3p_2 = 104 \\ -4p_1 + 4p_2 = 156 \end{cases}$$



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Solving Multi-Product Equilibrium

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$$(4p_1 - 4p_1) + (-3p_2 + 4p_2) = 104 + 156 \implies p_2 = \$260$$



Solving Multi-Product Equilibrium

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Solving Multi-Product Equilibrium

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Substitute $p_2 = 260$ back into Eq 1:

$$4p_1 - 3(260) = 104 \implies 4p_1 - 780 = 104 \implies 4p_1 = 884 \implies p_1 = \$221$$



Solving Multi-Product Equilibrium

Step 3: Solve the 2×2 Linear System

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Substitute $p_2 = 260$ back into Eq 1:

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Equilibrium Quantities:

$$q_1 = 2(221) - 4 = \mathbf{438} \text{ units}, \quad q_2 = 3(260) - 6 = \mathbf{774} \text{ units.}$$



Piecewise Linear Tax Structure

Joint Federal Income Tax Bracket Model (Example 10):

$$T = f(x) = \begin{cases} 0.15x, & 0 < x \leq 32,450 \\ 0.28x - 4,218.50, & 32,450 < x \leq 78,400 \\ 0.33x - 8,138.50, & 78,400 < x \leq 162,770 \\ 0.28x, & x > 162,770 \end{cases}$$



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Tax Computation for Income $x = \$50,000$ (Bracket 2):

Note: Marginal tax rate rises from 15% to 28% and 33% before returning to 28% due to phase-out rules.



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$$T = 0.28(50,000) - 4,218.50 = 14,000 - 4,218.50 = \mathbf{\$9,781.50}$$



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Discussions

Active Learning Discussions

Discussion

Discussion 1: Linear Assumptions vs. Real-World Economies of Scale

Why do financial managers rely heavily on linear cost models $C(x) = vx + FC$ even when bulk material discounts reduce unit costs at higher production volumes?



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Discussion

Discussion 2: Cross-Price Elasticity in Multi-Product Demand

In Example 9, $q_{d1} = 100 - 2p_1 + 3p_2$. What does the positive coefficient $+3p_2$ reveal about the relationship between Product 1 and Product 2?



Discussion

Discussion 3: Supply Curve Threshold Prices

If a manufacturer's supply curve is $q_s = 10p - 250$, what is the minimum market price required before the firm manufactures any units?



Summary & Homework

Summary & Homework Assignment

Lecture 07 Takeaways

- Total Cost $C(x) = vx + FC$; Total Revenue $R(x) = px$.
- Profit $P(x) = (p - v)x - FC$, where $(p - v)$ is marginal contribution.
- Straight-line depreciation distributes asset cost evenly across useful life.
- Demand curves slope downward ($m < 0$); Supply curves slope upward ($m > 0$).



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Homework Problems (Chapter 5)

- **Section 5.1:** Exercises 1, 4, 5, 7, 8
- **Section 5.2:** Exercises 1, 2, 7, 8, 9, 10, 11, 15, 19

Next class (Lecture 8): **Break-Even Models & Contribution Margin Analysis.**

