



Lecture 08

Fundamentals of Business Mathematics

MATH-114 • BBA 2k26

Dr. Rizwan Jahangir

NUST Business School

National University of Sciences & Technology

Lecture 08 Outline: Linear Functions — Applications & Break-Even

Part 1: Other Applications of Linear Functions

- Straight-line depreciation
- Depreciation with salvage value
- Linear demand and supply functions
- Restricted domains and interpretation

Part 2: Piecewise Linear Applications

- Tax-liability functions
- Interpreting different income ranges

Part 3: Break-Even Models

- Break-even assumptions
- Break-even condition
- Break-even quantity
- Profit and loss regions
- Graphical interpretation

Part 4: Practice & Summary

- Integrated business problems
- Key takeaways
- Textbook practice



Other Applications of Linear Functions

From Linear Functions to Business Applications

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- The intercept a_0 gives the value of y when $x = 0$, when that interpretation is meaningful.
- The practical domain and range may be restricted by the business situation.

Model first, interpret second: A mathematically valid linear function may still require a restricted domain in the business context.



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- Therefore, V is a linear function of t .
- If an asset costing C is fully depreciated after L years with no salvage value, then

$$V(t) = C - \frac{C}{L}t, \quad 0 \leq t \leq L.$$



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The negative slope represents the annual decrease in book value.



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Therefore,

$$V(t) = C - \frac{C - S}{L}t, \quad 0 < t < L.$$



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Book-value function:

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Check:

$$V(0) = 20,000, \quad V(5) = 2,500.$$

The function reaches the salvage value—not necessarily zero—at the end of the useful life.



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- The domain and range should be restricted to economically meaningful values.



Example 3: Constructing a Linear Demand Function

Problem: Two points on a linear demand function are

$$(\$20, 80, 000) \quad \text{and} \quad (\$30, 62, 500).$$

Determine the demand function $q = f(p)$.



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Using $(20, 80,000)$:

$$80,000 = -1,750(20) + b \quad \Rightarrow \quad b = 115,000.$$



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$$\boxed{q = f(p) = 115,000 - 1,750p}$$



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For practical use, restrict the domain to values of p for which the predicted quantity is meaningful.



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Use $q = mp + b$ and $(4, 28,000)$:

$$28,000 = 10,800(4) + b \quad \Rightarrow \quad b = -15,200.$$



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Piecewise Linear Applications

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Do not apply a marginal tax rate to the entire income when the model specifies that the rate applies only to income within a particular bracket.



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For the third range:

$$T = 0.33x - 8,138.5, \quad 78,400 < x \leq 162,770.$$



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For income above \$162,770:

$$T = 0.28x.$$



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Modeling lesson: Identify the relevant interval before selecting the formula.



Break-Even Models

Break-Even Analysis: The Basic Idea

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- Break-even analysis can be used as a planning tool for evaluating profitability.



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If price per unit is less than variable cost per unit, a break-even condition may never exist within the model.



Break-Even Analysis: Three Useful Measures

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In this lecture, we focus primarily on the **break-even output level**.



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In this lecture, we focus primarily on the **break-even output level**.

- Let x = level of output.
- $C(x)$ = total cost function.
- $R(x)$ = total revenue function.
- $P(x)$ = profit function.



Break-Even Procedure



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$$R(x) = C(x)$$

- 4 Solve for x . The resulting value is the **break-even level of output**, denoted x_{BE} .



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- 2 Formulate total revenue as a function of x .

$$R(x) = \text{selling price per unit} \times x.$$

- 3 Set total revenue equal to total cost.

$$R(x) = C(x)$$

- 4 Solve for x . The resulting value is the **break-even level of output**, denoted x_{BE} .

Alternative: Form the profit function $P(x) = R(x) - C(x)$, then solve $P(x) = 0$.

Example 6: Break-Even Analysis

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$$7.50x = 250,000 \quad \Rightarrow \quad x_{BE} = 33,333.33 \text{ units.}$$



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From the smoke-detector model,

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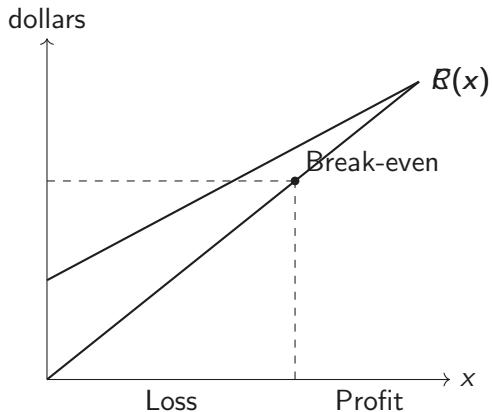
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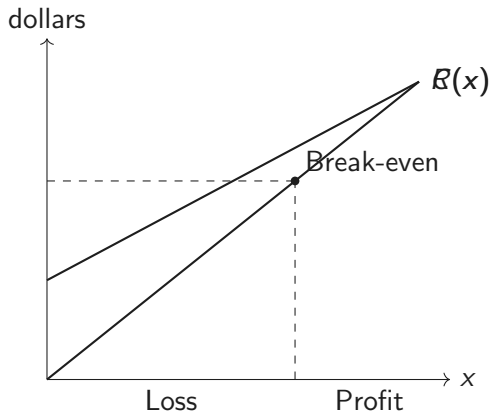
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The firm is expected to incur a **\$25,000 loss** at an output of 30,000 units.

Graphical Interpretation of Break-Even



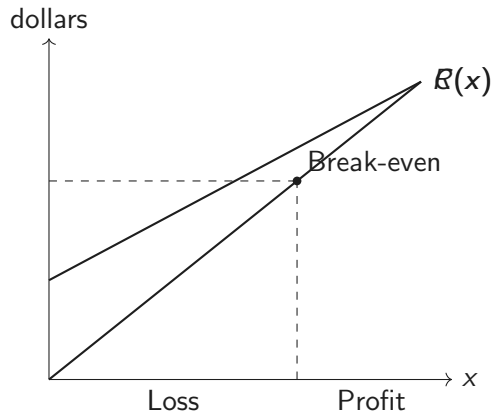
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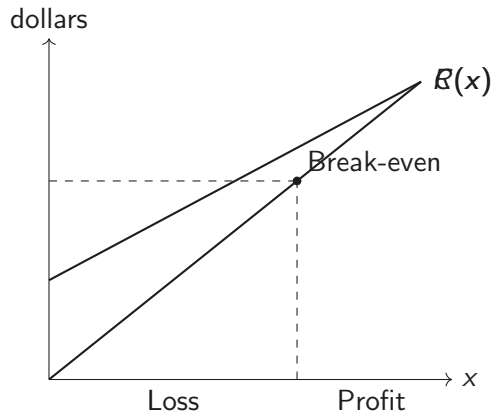
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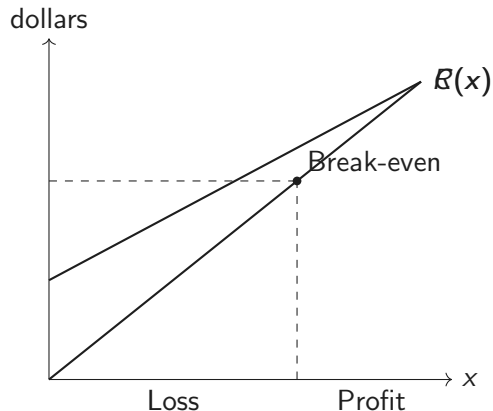
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At break-even, $R(x_{BE}) = C(x_{BE})$ and therefore $P(x_{BE}) = 0$.



Integrated Applications & Discussion

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Discussion

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Business translation is the key step: first identify the variables and relationship, then choose the mathematical form.



Break-Even Interpretation: Managerial Questions

Discussion

Suppose a firm's break-even output is 40,000 units.



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- 2 What does producing and selling more than 40,000 units imply?
- 3 What happens to the break-even output if fixed costs increase while all other parameters remain unchanged?
- 4 Why is it important to distinguish a mathematical break-even point from a practically feasible output level?



Summary & Practice

Lecture 08 Summary

Other Linear Applications

- Straight-line depreciation
- Salvage value and restricted domain
- Linear demand functions
- Linear supply functions
- Piecewise linear tax models

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- $R(x) = C(x)$ at break-even
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- Below break-even: loss
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Core modeling skill: translate a business situation into a linear function, respect the practical domain, and interpret the result in business terms.



Textbook Practice

Budnick, Applied Mathematics for Business, Economics, and the Social Sciences, 4th Ed.



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- **Section 5.3:** Practice the break-even examples and follow-up exercises assigned in the textbook.

These exercises reinforce the applications developed in Lecture 08 and prepare students for the Week 4 Assignment / Class Activity.



Transition to Lecture 09

Next Topic: Quadratic Functions

- Linear models use a constant rate of change.



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- The next chapter introduces quadratic functions, whose rate of change is not constant.
- We will examine their characteristics, graphical representation, and business applications.

For Lecture 08: be able to formulate, solve, and interpret linear business models before moving to quadratic models.

