



Lecture 10

Fundamentals of Business Mathematics

MATH-114 • BBA 2k26

Dr. Rizwan Jahangir

NUST Business School

National University of Sciences & Technology

Lecture 10 Outline

- Quadratic revenue functions
- Quadratic supply and demand functions
- Market equilibrium with quadratic functions
- A quadratic location model
- Additional business applications and practice



Quadratic Functions in Business Applications

A quadratic function has the form

$$f(x) = ax^2 + bx + c, \quad a \neq 0.$$

Its graph is a parabola. If $a < 0$, it opens downward; if $a > 0$, it opens upward.

- Revenue can be quadratic when quantity demanded depends linearly on price.
- Supply or demand data may be modelled by a quadratic function when three data points are available.
- Equilibrium is found by setting quantity supplied equal to quantity demanded.
- A quadratic objective can model a location decision when squared distances are used.



Quadratic Revenue

Example 1: Quadratic Revenue Function

Suppose that the demand function for a product is

$$q = f(p) \quad \text{or} \quad q = 1,500 - 50p,$$

where q equals the quantity demanded in thousands of units and p equals the price in dollars. Total revenue R from selling q units is stated as the product of p and q , or

$$R = pq.$$

Because the demand function states q as a function of p , total revenue can be stated as a function of price, or

$$\begin{aligned} R &= h(p) \\ &= p \cdot f(p) \\ &= p(1,500 - 50p) \\ &= 1,500p - 50p^2. \end{aligned}$$



Example 1: Solution

$$\begin{aligned} R &= pq \\ &= p(1,500 - 50p) \\ &= 1,500p - 50p^2. \end{aligned}$$

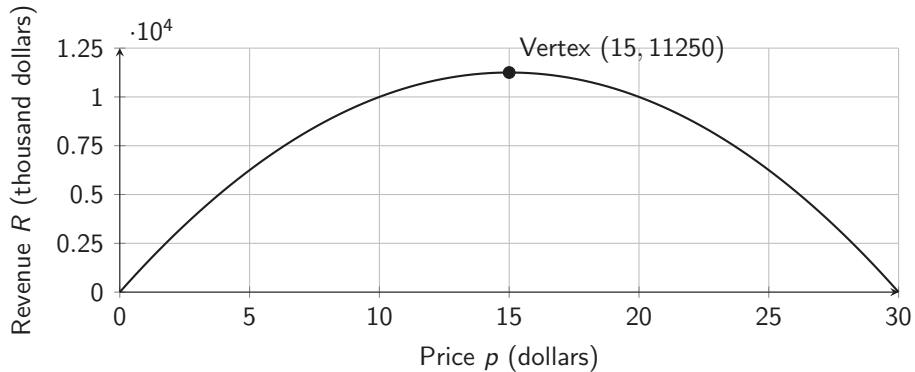
- This is a quadratic function with $a = -50 < 0$, so the parabola opens downward.
- The economically meaningful domain is $0 \leq p \leq 30$, because quantity demanded must be nonnegative.
- The maximum revenue occurs at the vertex.

$$p_{\max} = -\frac{b}{2a} = -\frac{1,500}{2(-50)} = 15.$$

$$R(15) = 1,500(15) - 50(15)^2 = 11,250$$

Thus the maximum is \$11.25 million when price is \$15 and quantity demanded is 750 thousand units.

Example 9: Graph of the Revenue Function



Quadratic Supply and Demand

Example 2: Quadratic Supply Functions

Market surveys of suppliers of a particular product have resulted in the conclusion that the supply function is approximately quadratic in form. Suppliers were asked what quantities they would be willing to supply at different market prices. Results of the survey indicated that at market prices of \$25, \$30, and \$40 the quantities which suppliers would be willing to offer to the market were 112.5, 250.0, and 600.0 (thousand) units, respectively.



Example 2: Solution

Assume the quadratic supply function is

$$q_s = ap^2 + bp + c.$$

Substitute the three price–quantity pairs:

$$625a + 25b + c = 112.5,$$

$$900a + 30b + c = 250,$$

$$1600a + 40b + c = 600.$$

Solving the system gives $a = 0.5$, $b = 0$, and $c = -200$. Hence

$$q_s = 0.5p^2 - 200.$$

The model is an estimate based on the three supplied observations. Interpret predictions only over a meaningful price range.

Using the Supply Function

Using $q_s = 0.5p^2 - 200$, estimate supply at $p = \$50$:

$$\begin{aligned}q_s(50) &= 0.5(50)^2 - 200 \\&= 1,250 - 200 \\&= 1,050.\end{aligned}$$

Estimated quantity supplied: 1,050 thousand units.



Using the Supply Function

Using $q_s = 0.5p^2 - 200$, estimate supply at $p = \$50$:

$$\begin{aligned}q_s(50) &= 0.5(50)^2 - 200 \\&= 1,250 - 200 \\&= 1,050.\end{aligned}$$

Estimated quantity supplied: 1,050 thousand units.

Discussion

What is the restricted domain of this supply function? Is it the same as the domain suggested by the graph? Interpret the q_s -intercept. Does this interpretation make sense?



Example 3: Quadratic Demand Functions

Related to the previous example, a consumer survey was conducted to determine the demand function for the same product. Researchers asked consumers if they would purchase the product at various prices and from their responses constructed estimates of market demand at various market prices. After sample data points were plotted, it was concluded that the demand relationship was best represented by a quadratic function. Researchers concluded that the quadratic representation was valid for prices between \$5 and \$45.

Three data points chosen for “fitting” the curve were (5, 2,025), (10, 1,600), and (20, 900). Substituting these data points into the general equation for a quadratic function and solving the resulting system simultaneously gives the demand function.



Example 3: Solution

Assume

$$q_d = ap^2 + bp + c.$$

Using the points (5, 2025), (10, 1600), and (20, 900) gives

$$25a + 5b + c = 2025,$$

$$100a + 10b + c = 1600,$$

$$400a + 20b + c = 900.$$

Solving gives $a = 1$, $b = -100$, and $c = 2500$. Therefore,

$$\boxed{q_d = p^2 - 100p + 2,500}, \quad 5 \leq p \leq 45.$$



Practice Exercise: Quadratic Demand

At a price of \$30, how many units are expected to be demanded?



Practice Exercise: Quadratic Demand

At a price of \$30, how many units are expected to be demanded?

$$\begin{aligned}q_d(30) &= 30^2 - 100(30) + 2,500 \\&= 900 - 3,000 + 2,500 \\&= 400.\end{aligned}$$

Answer: 400 thousand units.



Market Equilibrium

Example 4: Supply–Demand Equilibrium

Market equilibrium between supply and demand can be estimated for the supply and demand functions in the last two examples by determining the market price which equates quantity supplied and quantity demanded. This equilibrium condition is expressed by the equation

$$q_s = q_d.$$

If we substitute the supply and demand functions from Examples 10 and 11 into this equation, we get

$$0.5p^2 - 200 = p^2 - 100p + 2,500.$$



Example 4: Solution

Start with

$$0.5p^2 - 200 = p^2 - 100p + 2,500.$$

Rearrange:

$$0.5p^2 - 100p + 2,700 = 0.$$

Using the quadratic formula,

$$\begin{aligned} p &= \frac{100 \pm \sqrt{(-100)^2 - 4(0.5)(2,700)}}{2(0.5)} \\ &= 100 \pm \sqrt{4,600} \\ &\approx 100 \pm 67.82. \end{aligned}$$

The roots are approximately $p = \$32.18$ and $p = \$167.82$. Only $\$32.18$ lies in the demand model's domain $5 \leq p \leq 45$.



Example 4: Interpretation

At $p \approx \$32.18$,

$$q_s = 0.5(32.18)^2 - 200 \approx 317.77,$$

$$q_d = (32.18)^2 - 100(32.18) + 2,500 \approx 317.55.$$

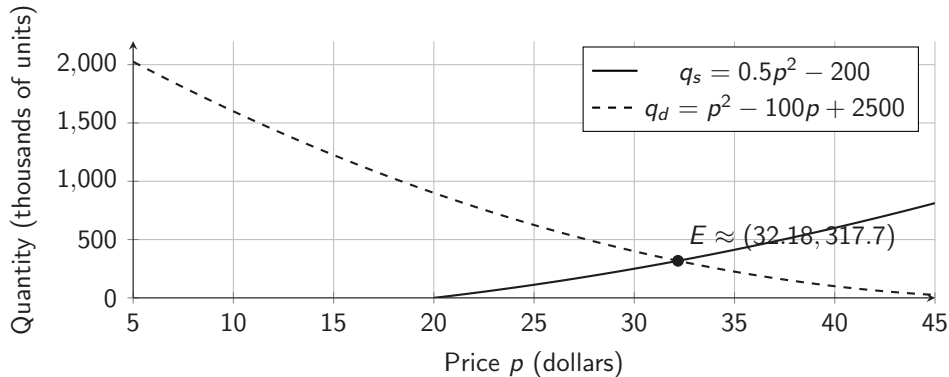
The slight difference is due to rounding. Thus the estimated market equilibrium is approximately:

- **Equilibrium price:** \$32.18
- **Equilibrium quantity:** 317.77 thousand units (approximately)

Solve $q_s = q_d$, then reject any root outside the relevant domain of the models.



Supply and Demand: Graphical Interpretation



The intersection within the demand model's stated domain gives the estimated equilibrium.



Quadratic Location Model

Example 5: Emergency Response Location Model

Problem statement

Figure 6.13 illustrates the relative locations of three cities along a coastal highway. The three cities are popular resorts, and their populations swell during the summer months. The three cities believe their emergency rescue and health treatment capabilities are inadequate during the vacation season. They have decided to support jointly an emergency response facility which dispatches rescue trucks and trained paramedics. A key question concerns the location of the facility.

In choosing the location, it has been agreed that the distance from the facility to each city should be kept as short as possible to ensure quick response times. Another consideration is the size of the summer population of each city, since this is one measure of the potential need for emergency response services. The larger the summer population of a city, the greater the desire to locate the facility close to the city. Analysts have decided that the criterion for selecting the location is to minimize the sum of the products of the summer populations of each town and the square of the distance between the town and the facility.



Example 5: Forming the Objective Function

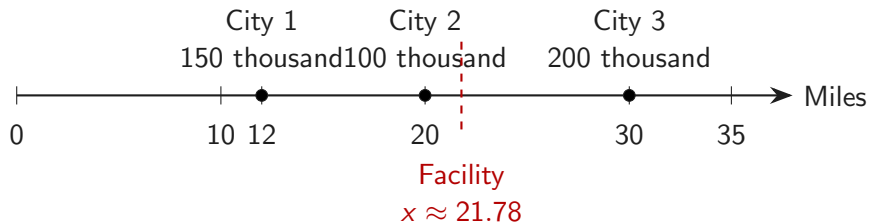
The criterion is given as

$$\text{Minimize } S = \sum_{j=1}^3 p_j d_j^2.$$

If the summer populations are, respectively, 150,000, 100,000, and 200,000 for the three cities, compute the general expression for S . (Hint: Let x equal the location of the facility relative to the zero point of the scale in Fig. 6.13, and let x_j equal the location of city j . The distance between the facility and city j is calculated by the equation $d_j = x - x_j$.)



Example 5: Relative Locations Along the Highway



The model minimizes the population-weighted sum of squared distances, not simply the unweighted sum of distances.



Example 5: Solution

The cities are located at $x_1 = 12$, $x_2 = 20$, and $x_3 = 30$ miles. Populations are expressed in thousands, so

$$\begin{aligned} S &= 150(x - 12)^2 + 100(x - 20)^2 + 200(x - 30)^2 \\ &= 150(x^2 - 24x + 144) + 100(x^2 - 40x + 400) \\ &\quad + 200(x^2 - 60x + 900) \\ &= 450x^2 - 19,600x + 241,600. \end{aligned}$$

Since $a = 450 > 0$, the parabola opens upward and S is minimized at its vertex:

$$x = -\frac{b}{2a} = \frac{19,600}{900} \approx 21.78.$$

Conclusion: Locate the facility approximately 21.78 miles from the zero point, or about 1.78 miles to the right of City 2.



Follow-up Exercise

Follow-up Exercise 13

Problem statement (Section 6.2 Follow-up Exercises, Exercise 13):

The supply and demand functions for a product are $q_s = p^2 - 400$ and $q_d = p^2 - 40p + 2,600$. Determine the market equilibrium price and quantity.



Follow-up Exercise 13

Problem statement (Section 6.2 Follow-up Exercises, Exercise 13):

The supply and demand functions for a product are $q_s = p^2 - 400$ and $q_d = p^2 - 40p + 2,600$. Determine the market equilibrium price and quantity.

Solution: Set $q_s = q_d$:

$$\begin{aligned}p^2 - 400 &= p^2 - 40p + 2,600 \\-400 &= -40p + 2,600 \\40p &= 3,000 \\p &= 75.\end{aligned}$$

Substitute into either function:

$$q = 75^2 - 400 = 5,225.$$

Equilibrium: price = \$75; quantity = 5,225 units.



Suggested Problems

Suggested Problems: Section 6.2 Follow-up Exercises

- Quadratic revenue functions: 1–6
- Quadratic supply functions: 7–9
- Quadratic demand functions: 10–12
- Market equilibrium: 13–14
- Location models: 15–17
- Quadratic modelling and prediction: 18–19



Additional Practice

Additional Practice 1: Revenue and Break-Even Quantity

Problem 1. A campus shop estimates that when it charges p dollars per notebook, weekly demand is $q = 1,200 - 40p$ notebooks. Assume the model is valid for $0 \leq p \leq 30$.

- 1 Express weekly revenue R as a function of price p .
- 2 Find the price that maximizes weekly revenue.
- 3 Determine the maximum weekly revenue and the corresponding quantity sold.

Problem 2. A supplier's quantity supplied is estimated by $q_s = 0.2p^2 - 50$, while demand is estimated by $q_d = 700 - 10p$.

- 1 Find the market equilibrium price and quantity.
- 2 State which root(s) are economically meaningful, if the equation produces more than one root.



Additional Practice 2: Quadratic Business Models

Problem 3. A firm estimates that its weekly revenue, in thousands of dollars, is

$$R(x) = -2x^2 + 160x,$$

where x is the selling price in dollars and the model is valid for $0 \leq x \leq 80$.

- 1 Find the price that maximizes revenue.
- 2 Calculate the maximum revenue.
- 3 Find the price(s) at which the model predicts zero revenue.

Problem 4. A delivery company is choosing a depot location x along a straight road. Its weighted-distance cost index is estimated by

$$C(x) = 3x^2 - 96x + 900,$$

where x is measured in kilometres from a fixed reference point.

- 1 Find the location that minimizes $C(x)$.
- 2 Calculate the minimum value of the cost index.

