



Lecture 12

Fundamentals of Business Mathematics

MATH-114 • BBA 2k26

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Lecture 12 Outline

- Section 7.2: Applications of exponential functions
- Compound interest, continuous compounding, growth, and decay
- Section 7.3: Logarithms and logarithmic functions
- Logarithm properties and solving exponential/logarithmic equations
- Suggested textbook problems and additional practice



Applications of Exponential Functions

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- **Growth:** population, investment value, and other quantities that increase by a constant percentage.
- **Decay:** depreciation, drug excretion, and other quantities that decrease by a constant percentage.
- The model and its parameters depend on the context and the units used.



Example

The equation

$$S = P(1 + i)^n$$

determines the amount S to which an investment of P dollars grows when it receives interest at rate i per compounding period for n compounding periods, assuming reinvestment of accrued interest.



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$$\begin{aligned} S &= 1000(1.08)^{25} \\ &\approx 1000(6.8485) = \$6,848.50. \end{aligned}$$



Example

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$$\begin{aligned} S &= 1000e^{0.08(25)} = 1000e^2 \\ &\approx 1000(7.3891) = \$7,389.10. \end{aligned}$$



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$$P(t) = P_0 e^{kt} = 100e^{0.04t}$$

$$t = 25 \quad \text{for the year 1995}$$

$$\begin{aligned} P(25) &= 100e^{0.04(25)} = 100e \\ &\approx 271.83 \text{ million.} \end{aligned}$$



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For $V(t) = V_0 e^{kt}$, V_0 is the initial value and $k > 0$ is the continuous growth rate.



Practice: Constant Percentage Growth

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$$\begin{aligned}\frac{f(t+1)}{f(t)} &= \frac{V_0 e^{k(t+1)}}{V_0 e^{kt}} \\ &= e^k.\end{aligned}$$

The ratio is constant, independent of t . Thus, each one-unit increase multiplies the current value by the same factor e^k , giving a constant percentage increase.



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An exponential decay process is characterized by a constant percentage decrease in value over time and may be modeled by

$$V(t) = V_0 e^{-kt}.$$

The resale value of a type of industrial equipment is

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$$V(0) = 100,000e^0 = \$100,000$$

$$V(5) = 100,000e^{-0.5} \approx \$60,650$$

$$V(10) = 100,000e^{-1} \approx \$36,790.$$



Example

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- As $t \rightarrow \infty$, $P(t) \rightarrow 0.95$; the model approaches 95%.



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- 2 Find the amount after 8 hours, 10 hours, and 24 hours.



Solution: Prescription Drug Excretion

Each 4-hour period leaves one-half of the amount from the previous period. Therefore,

$$A(t) = 300 \left(\frac{1}{2} \right)^t$$

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8 hours: $t = 2$,

$$A(2) = 300(0.5)^2 = 75 \text{ mg}$$

10 hours: $t = 2.5$,

$$A(2.5) = 300(0.5)^{2.5} \approx 53.03 \text{ mg}$$

24 hours: $t = 6$,

$$A(6) = 300(0.5)^6 \approx 4.69 \text{ mg}.$$



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The graph is an exponential decay curve, beginning at $A(0) = 300$ and approaching zero.



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The common logarithm has base 10: $\log x = \log_{10} x$. The natural logarithm has base e : $\ln x = \log_e x$.



Example

Write each statement in its equivalent form.

Logarithmic form	Exponential form
$\log_2 16 = 4$	$2^4 = 16$
$\log_{10} 100 = 2$	$10^2 = 100$
$\log_3 27 = 3$	$3^3 = 27$
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Conversely,

$$10^4 = 10,000 \iff \log_{10} 10,000 = 4.$$

$$e^3 = x \iff \ln x = 3.$$



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A logarithm converts products to sums, quotients to differences, and powers to coefficients. It does not distribute over addition: $\log_b(u + v) \neq \log_b u + \log_b v$ in general.

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Solve the logarithmic equation

$$\ln x^2 + \ln x = 9.$$



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Since $\ln x$ is defined only for $x > 0$, the solution must be positive. Using the power property,

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Thus, the solution is $x \approx 20.086$.



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Using $e^2 \approx 7.3891$,

$$7.3891x^2 = x^2 + 2$$

$$6.3891x^2 = 2$$

$$x^2 \approx 0.3130$$

$$x \approx \pm 0.5595.$$



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Then

$$\ln C_0 = \ln 800 - 25(0.1001) \approx 4.1821, \quad C_0 \approx 65.5033.$$

$$C(t) \approx 65.5033e^{0.1001t}$$



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If $k = 0.4$ and time is measured in hours,

$$t = \frac{0.6932}{0.4} \approx 1.733 \text{ hours.}$$



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For strontium-90, $k = 0.0244$ when t is measured in years:

$$t = \frac{-0.6932}{-0.0244} \approx 28.40 \text{ years.}$$



Example

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The model decreases as t increases over its stated domain.



Suggested Textbook Problems

Section 7.2: 1–5, 8–12, 16–18, 22–25, 47

Section 7.3: 1–24, 25–40, 41–60, 61–80, 81–92



Additional Practice

Additional Practice: Business and Science Applications

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- 3 **Customer collections:** The fraction of an invoice collected after t months is $P(t) = 0.98(1 - e^{-0.45t})$. Find the percentage collected after 3 months and the limiting percentage.



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- 3 **Customer collections:** The fraction of an invoice collected after t months is $P(t) = 0.98(1 - e^{-0.45t})$. Find the percentage collected after 3 months and the limiting percentage.
- 4 **Bacterial growth:** A culture follows $N(t) = N_0 e^{0.3t}$. Determine the doubling time, and state how it changes if the growth constant becomes 0.6.



Additional Practice: Solutions

1. Investment

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3. Collections

$$P(3) = 0.98(1 - e^{-1.35}) \approx 0.7259 = 72.59\%.$$

The limiting percentage is 98%.



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The limiting percentage is 98%.

4. Doubling time

$$t = \frac{\ln 2}{0.3} \approx 2.31 \text{ time units}; \quad t = \frac{\ln 2}{0.6} \approx 1.16 \text{ time units}.$$

Doubling time is halved when the growth constant doubles.

