



Lecture 13

Fundamentals of Business Mathematics

MATH-114 • BBA 2k26

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Lecture 13: Learning Goals

By the end of this lecture, you should be able to:

- explain the meaning of principal, interest, interest rate, and time period;
- calculate simple interest and the total amount due;
- distinguish simple interest from compound interest;
- calculate a compound amount using a formula or a period-by-period table;
- explain why reinvesting interest makes compound growth powerful over time.



8.1 Interest and Its Computation

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The interest rate and the number of periods must use consistent units. If the rate is annual, time should be measured in years.



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Unit check

If the rate is 9% per year and the loan lasts 18 months, use $i = 0.09$ and $n = 1.5$ years.



Example 1: A Three-Year Loan

Question. A credit union issues a 3-year loan of \$5,000. Simple interest is charged at 10% per year. The principal plus interest is repaid at the end of the third year. Compute the interest for the 3-year period and the amount repaid at the end of the third year.



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Step 3: Add the principal.

$$A = P + I = 5000 + 1500 = \boxed{\$6,500}.$$



Example 2: Interest Paid Each Quarter

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Do not multiply the 3% quarterly rate by 5 years directly. First express the duration in quarters, or calculate interest per quarter and multiply by 20.



The Power of Compound Growth

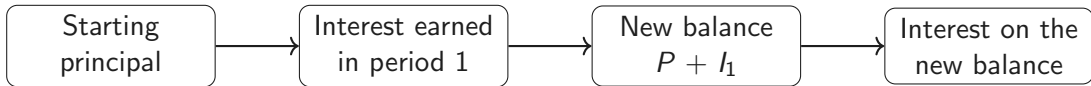
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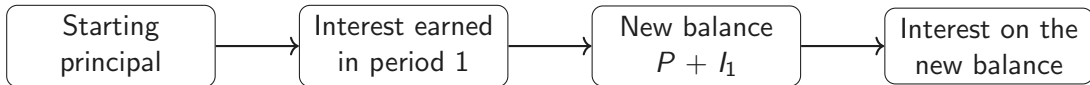
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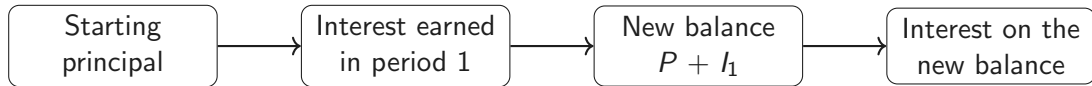


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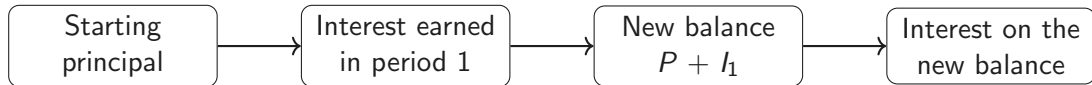


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- Compound interest earns interest on both the original principal and accumulated interest.
- The difference becomes more noticeable over longer periods and at higher rates.



Compound Interest: Period-by-Period Calculation

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Each new period begins with the previous period's accumulated amount, not the original \$8,000.



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Difference:

$$8659.46 - 8640 = \boxed{\$19.46}.$$



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Common compounding frequencies:

Frequency	Periods per year (m)
Annually	1
Semiannually	2
Quarterly	4
Monthly	12
Daily (often approximated)	365



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The accumulated interest is approximately

$$15,245.88 - 12,000 = \$3,245.88.$$



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A small difference in annual growth can become substantial when it is repeatedly applied to an increasing balance.



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- Over 20 years, the balance grows by about \$14,348.42.
- The 20-year balance is not merely four times the 5-year interest gain: the growth compounds on itself.



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- 4 Does the time need conversion to match the rate or compounding period?
- 5 For compound interest, how many compounding periods are there?

Simple interest: $I = Pin$ and $A = P + I$.

Compound amount: $A = P(1 + i/m)^{mt}$.



Suggested Problems: Section 8.1

Textbook Practice: Interest and Its Computation

Core Practice

- Exercises 1–2: Simple interest and total repayment
- Exercise 3: Semiannual simple interest on corporate bonds
- Exercises 5–6: Compound interest using tables
- Exercise 4: Simple interest in a business-loan context
- Exercise 8: Comparing semiannual and quarterly compounding
- Exercise 7: Comparing different compounding frequencies



Additional Practice: Try Before Viewing the Answers

- 1 A \$7,500 loan charges simple interest at 9% per year for 2.5 years. Find the interest and total repayment
- 2 A \$20,000 deposit earns 5.4% per year compounded semiannually. Find the amount after 6 years
- 3 A company invests \$50,000 at 7.2% per year compounded monthly for 3 years. Calculate the accumulated amount and interest earned.
- 4 Two accounts each start with \$10,000 for 10 years. Account A earns 6% compounded annually; Account B earns 6% compounded monthly. Find both balances and compare them.



Additional Practice: Solutions

1. Simple interest

$$I = 7500(0.09)(2.5) = \$1,687.50, \quad A = \boxed{\$9,187.50}.$$

2. Semiannual compounding

$$A = 20000(1 + 0.054/2)^{12} \approx \boxed{\$27,534.38}.$$

3. Monthly compounding

$$A = 50000(1 + 0.072/12)^{36} \approx \boxed{\$62,015.08}.$$

Interest earned $\approx \$12,012.14$. **4. Annual versus monthly compounding**

$$A_A = 10000(1.06)^{10} \approx \$17,908.48,$$

$$A_B = 10000(1 + 0.06/12)^{120} \approx \$18,193.97.$$

Monthly compounding gives approximately **\$285.49** more.

