



Lecture 04

Statistics II

STAT-221 • UG

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Welcome, Review & Quiz 1

The Pivot: From Estimation to Testing

For the past two weeks, our core problem has been **Estimation**.

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- So, we took a sample, found the point estimate ($\bar{x}$), and built a Confidence Interval around it to "capture" the true value.

Today, we reverse the process!

In Chapter 10, someone is making a **specific claim** about a population parameter. We will use sample data to test if that claim is mathematically reasonable, or if it is false.



Quiz 1 (15 Minutes)

Topics: Normal Distribution, Sampling, & Intervals

Introduction to Hypothesis Testing

Icebreaker: Real-World Claims

Discussion

Every day, businesses make claims that need to be tested:

- Dole Pineapple Inc. claims their sliced pineapple cans contain exactly 16 ounces.
- A university claims the mean starting salary for a business graduate is \$51,541.
- Chevy claims their TrailBlazer leases average 32,000 miles.

If you test 50 cans of Dole Pineapple and the average is 15.8 ounces, does that prove the company is lying? Or could it just be sampling error? Let's discuss.



What is Hypothesis Testing?

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Hypothesis Testing: A procedure based on sample evidence and probability theory to determine whether the hypothesis is a reasonable statement.



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The Six-Step Procedure:

- 1 State the Null (H_0) and Alternate (H_1) hypotheses.
- 2 Select a level of significance (α).
- 3 Select the test statistic.
- 4 Formulate the decision rule.
- 5 Make a decision.
- 6 Interpret the result.



Step 1: State the Hypotheses

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H_0 and H_1 are **mutually exclusive** and **collectively exhaustive**. If one is true, the other must be false.



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Type I Error (False Positive):

- Rejecting a true null hypothesis.
- The probability of this happening is exactly α (usually 0.05 or 0.01).

Type II Error (False Negative):

- Accepting a false null hypothesis.
- The probability of this happening is β .



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LASIK Surgery Example: Surgery has a 5% complication rate. The null hypothesis (H_0) is that surgery will *not* improve eyesight. The 5% complication rate represents the **Type I error**: the cases where we rejected H_0 (we operated), but the expectation of improved eyesight was not met!



The Six-Step Procedure (Steps 3 to 6)

Steps 3 & 4: The Statistic and Decision Rule

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- In this lecture, since the population standard deviation (σ) is **known**, we will use the standard normal **z-statistic**:

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$



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- A statement of the specific conditions under which H_0 is rejected.
- We establish a **Critical Value**, which is the dividing point between the region where H_0 is rejected and where it is not rejected.



Steps 5 & 6: Decision and Interpretation

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Step 6: Interpret the Result

- Statistics is not just math; it is business decision-making.
- Simply writing "Reject H_0 " will earn you zero marks on the exam. You must write a sentence explaining what your mathematical decision means for the company!



15 Minute Break

Fully Solved Interactive Examples

Example 1: Jamestown Steel (Two-Tailed Test)

Textbook Example: Jamestown Steel Company

The weekly production of the Model A325 desk at the Fredonia plant follows a normal probability distribution with a mean of 200 and a population standard deviation (σ) of 16.

Recently, new production methods were introduced. The VP of manufacturing took a sample of 50 weeks ($n = 50$) and found the sample mean ($\bar{x}$) was 203.5 desks.

Task: Is the mean number of desks produced **different** from 200 at the .01 significance level?



Jamestown Steel: Steps 1 & 2

Step 1: State the Null and Alternate Hypotheses.



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- $H_0 : \mu = 200$
- $H_1 : \mu \neq 200$



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Step 2: Select the Level of Significance.

The problem states we must use the 1% level.

- $\alpha = 0.01$



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Step 4: Formulate the Decision Rule.

Since this is a two-tailed test with $\alpha = 0.01$:

- Split α in half: $0.01/2 = 0.005$ in each tail.
- Middle area is $0.5000 - 0.0050 = 0.4950$.
- Looking up 0.4950 in the z -table gives a critical value of ± 2.58 .



Jamestown Steel: Steps 3 & 4

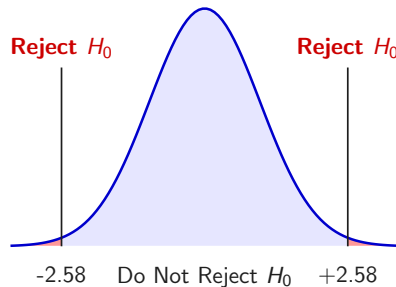
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Calculate the test statistic:

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$z = \frac{203.5 - 200}{16/\sqrt{50}}$$

$$z = \frac{3.5}{2.2627} = \mathbf{1.55}$$



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Because 1.55 does *not* fall in the rejection region (it is between -2.58 and 2.58), we **do not reject** H_0 .



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$$z = \frac{203.5 - 200}{16/\sqrt{50}}$$

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Because 1.55 does *not* fall in the rejection region (it is between -2.58 and 2.58), we **do not reject** H_0 .

Step 6: Interpret the Result.

We cannot reject the null hypothesis.

"The population mean is not different from 200. The difference of 3.5 desks between the historical mean and the sample mean can be attributed to sampling error. The VP cannot conclude that the new production methods changed the overall production rate."



Example 2: Dole Pineapple (One-Tailed Test)

Textbook Example: Dole Pineapple

Dole Pineapple Inc. is concerned that the 16-ounce can of sliced pineapple is being **overfilled**. Assume the population standard deviation (σ) is **.03 ounce**.

The quality control department took a random sample of **50 cans** ($n = 50$) and found that the sample mean ($\bar{x}$) was **16.05 ounces**.

Task: At the **5% level of significance**, can we conclude that the mean weight is **greater than 16 ounces**?



Dole Pineapple: Steps 1 to 4

1. Hypotheses (One-Tailed):

- $H_0 : \mu \leq 16$
- $H_1 : \mu > 16$ (Concern is overfilling)



Dole Pineapple: Steps 1 to 4

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- $H_0 : \mu \leq 16$
- $H_1 : \mu > 16$ (Concern is overfilling)

2. Significance Level: $\alpha = 0.05$



Dole Pineapple: Steps 1 to 4

1. Hypotheses (One-Tailed):

- $H_0 : \mu \leq 16$
- $H_1 : \mu > 16$ (Concern is overfilling)

2. Significance Level: $\alpha = 0.05$

3. Test Statistic: z (since $\sigma = 0.03$ is known).



Dole Pineapple: Steps 1 to 4



1. Hypotheses (One-Tailed):

- $H_0 : \mu \leq 16$
- $H_1 : \mu > 16$ (Concern is overfilling)

2. Significance Level: $\alpha = 0.05$

3. Test Statistic: z (since $\sigma = 0.03$ is known).

4. Decision Rule:

- Entire α is in the **right tail**.
- $0.5000 - 0.0500 = 0.4500$.
- Critical z -value = **1.65**.

Dole Pineapple: Steps 1 to 4

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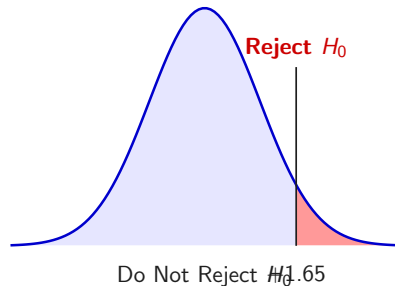
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$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$z = \frac{16.05 - 16}{0.03 / \sqrt{50}}$$

$$z = \frac{0.05}{0.00424} = \mathbf{11.79}$$



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11.79 is massively larger than our critical value of 1.65. It falls deep into the rejection region. We **reject** H_0 .



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11.79 is massively larger than our critical value of 1.65. It falls deep into the rejection region. We **reject** H_0 .

Step 6: Interpret the Result.

"We firmly reject the null hypothesis. The sample provides overwhelming evidence that the cans are indeed being overfilled above the 16-ounce target. The quality control department must immediately halt production and recalibrate the filling machines!"



Example 3: Exercise Application

Textbook Exercise 10 (Adapted)

Given the following hypotheses:

- $H_0 : \mu = 400$
- $H_1 : \mu \neq 400$

A random sample of **100 observations** is selected from a normal population with a known population standard deviation (σ) of **50**. The sample mean ($\bar{x}$) was **407**.

Task: Using the **.05 significance level**, conduct a full hypothesis test.

Group Task: Take 5 minutes to determine the critical value, compute the test statistic, and make a decision.



Example 3: Solution

Step 4: Decision Rule (Two-Tailed, $\alpha = 0.05$)

- Critical limits are $z = \pm 1.96$. Reject H_0 if computed $z < -1.96$ or $z > 1.96$.



Example 3: Solution

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$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{407 - 400}{50/\sqrt{100}}$$

$$z = \frac{7}{50/10} = \frac{7}{5} = \mathbf{1.40}$$



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$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{407 - 400}{50/\sqrt{100}}$$

$$z = \frac{7}{50/10} = \frac{7}{5} = \mathbf{1.40}$$

Step 6: Conclusion

- Since 1.40 falls between -1.96 and 1.96 , we **do not reject** H_0 . There is insufficient evidence to conclude the population mean is different from 400.

