



Lecture 05

Statistics II

STAT-221 • UG

Dr. Rizwan Jahangir

NUST Business School

National University of Sciences & Technology

Contents

- **1. Warm-up & Review (Translating Claims):**
 - ▶ Reviewing the Six-Step Procedure.
 - ▶ Translating English into Math (H_0 and H_1).
- **2. One-Tailed and Two-Tailed Hypothesis Tests:**
 - ▶ How H_1 dictates the direction.
 - ▶ Defining the Rejection Regions.
- **3. The p -Value in Hypothesis Testing:**
 - ▶ Measuring the exact strength of evidence.
 - ▶ The Universal Decision Rule.
- **4. Break (15 Minutes)**
- **5. Fully Solved p -Value Examples:**
 - ▶ Example 1: Dole Pineapple (One-Tailed).
 - ▶ Example 2: Crosset Trucking (Two-Tailed).
 - ▶ Example 3: Live Class Activity (Exercise 21).



Warm-up & Review (Translating Claims)

Review: The Pivot from Estimation to Testing

In Lecture 4, we learned that hypothesis testing is about verifying a **claim** made about a population parameter.



Review: The Pivot from Estimation to Testing

In Lecture 4, we learned that hypothesis testing is about verifying a **claim** made about a population parameter.

The Foundational Rule of Hypotheses:

- **Null Hypothesis (H_0):** The statement being tested. It **always** contains the equals sign ($=$, $\leq$, or $\geq$). We assume it is true until proven false.
- **Alternate Hypothesis (H_1):** The statement we accept if the sample data provides strong enough evidence. It **never** contains an equals sign ($\neq$, $>$, or $<$).



Review: The Pivot from Estimation to Testing

In Lecture 4, we learned that hypothesis testing is about verifying a **claim** made about a population parameter.

The Foundational Rule of Hypotheses:

- **Null Hypothesis (H_0):** The statement being tested. It **always** contains the equals sign ($=$, $\leq$, or $\geq$). We assume it is true until proven false.
- **Alternate Hypothesis (H_1):** The statement we accept if the sample data provides strong enough evidence. It **never** contains an equals sign ($\neq$, $>$, or $<$).

If H_1 uses a $\neq$ sign, it is a **Two-Tailed Test**.

If H_1 uses a $>$ or $<$ sign, it is a **One-Tailed Test**.



Interactive Translation: English to Math

Discussion

Let's practice translating business claims into mathematical hypotheses.

Identify H_0 , H_1 , and whether it is a One-Tailed or Two-Tailed test.

Scenario 1: "The Jamestown Steel Company historically produced 200 desks per week. Has the new production method **changed** the production rate?"



Interactive Translation: English to Math

Discussion

Let's practice translating business claims into mathematical hypotheses.

Identify H_0 , H_1 , and whether it is a One-Tailed or Two-Tailed test.

Scenario 1: "The Jamestown Steel Company historically produced 200 desks per week. Has the new production method **changed** the production rate?"

- "Changed" means it could be higher or lower (different).
- $H_0 : \mu = 200$
- $H_1 : \mu \neq 200$ (**Two-Tailed Test**)



Interactive Translation: English to Math

Scenario 2: "Dole Pineapple claims their cans contain 16 ounces. The quality control department is worried the machines are **overfilling** the cans."



Interactive Translation: English to Math

Scenario 2: "Dole Pineapple claims their cans contain 16 ounces. The quality control department is worried the machines are **overfilling** the cans."

- "Overfilling" specifies a clear direction (*greater than*).
- $H_0 : \mu \leq 16$
- $H_1 : \mu > 16$ (**One-Tailed Test, Right Tail**)



Interactive Translation: English to Math

Scenario 2: "Dole Pineapple claims their cans contain 16 ounces. The quality control department is worried the machines are **overfilling** the cans."

- *"Overfilling" specifies a clear direction (greater than).*
- $H_0 : \mu \leq 16$
- $H_1 : \mu > 16$ (**One-Tailed Test, Right Tail**)

Scenario 3: "A diet program claims that participants will lose **at least** 10 pounds in the first month. We want to prove they lose **less** than 10."



Interactive Translation: English to Math

Scenario 2: "Dole Pineapple claims their cans contain 16 ounces. The quality control department is worried the machines are **overfilling** the cans."

- "Overfilling" specifies a clear direction (greater than).
- $H_0 : \mu \leq 16$
- $H_1 : \mu > 16$ (**One-Tailed Test, Right Tail**)

Scenario 3: "A diet program claims that participants will lose **at least** 10 pounds in the first month. We want to prove they lose **less** than 10."

- "Less than" specifies a clear direction.
- $H_0 : \mu \geq 10$
- $H_1 : \mu < 10$ (**One-Tailed Test, Left Tail**)



One-Tailed and Two-Tailed Hypothesis Tests

Visualizing the Tails: Two-Tailed Test

When H_1 has a $\neq$ sign, we do not know which direction the error might be. Therefore, we must split our risk (Significance Level, α) evenly into **both** tails.

Example: $\alpha = 0.05$



Visualizing the Tails: Two-Tailed Test

When H_1 has a $\neq$ sign, we do not know which direction the error might be. Therefore, we must split our risk (Significance Level, α) evenly into **both** tails.

Example: $\alpha = 0.05$

- We split α : $0.05/2 = \mathbf{0.025}$ in each tail.



Visualizing the Tails: Two-Tailed Test

When H_1 has a $\neq$ sign, we do not know which direction the error might be. Therefore, we must split our risk (Significance Level, α) evenly into **both** tails.

Example: $\alpha = 0.05$

- We split α : $0.05/2 = \mathbf{0.025}$ in each tail.
- The middle area is $0.5000 - 0.0250 = \mathbf{0.4750}$.



Visualizing the Tails: Two-Tailed Test

When H_1 has a $\neq$ sign, we do not know which direction the error might be. Therefore, we must split our risk (Significance Level, α) evenly into **both** tails.

Example: $\alpha = 0.05$

- We split α : $0.05/2 = \mathbf{0.025}$ in each tail.
- The middle area is $0.5000 - 0.0250 = \mathbf{0.4750}$.
- Look up 0.4750 in the z-table $\rightarrow$ **Critical Values:** ± 1.96 .

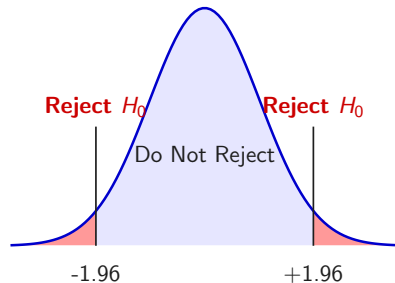


Visualizing the Tails: Two-Tailed Test

When H_1 has a $\neq$ sign, we do not know which direction the error might be. Therefore, we must split our risk (Significance Level, α) evenly into **both** tails.

Example: $\alpha = 0.05$

- We split α : $0.05/2 = \mathbf{0.025}$ in each tail.
- The middle area is $0.5000 - 0.0250 = \mathbf{0.4750}$.
- Look up 0.4750 in the z-table $\rightarrow$ **Critical Values:** ± 1.96 .



Visualizing the Tails: One-Tailed Test

When H_1 specifies a direction ($>$ or $<$), we place the **entire** risk (Significance Level, α) into just **one** tail.

Example: $\alpha = 0.05$ (Right-Tailed)



Visualizing the Tails: One-Tailed Test

When H_1 specifies a direction ($>$ or $<$), we place the **entire** risk (Significance Level, α) into just **one** tail.

Example: $\alpha = 0.05$ (Right-Tailed)

- We do **not** split α . The entire 0.05 goes in the right tail.



Visualizing the Tails: One-Tailed Test

When H_1 specifies a direction ($>$ or $<$), we place the **entire** risk (Significance Level, α) into just **one** tail.

Example: $\alpha = 0.05$ (Right-Tailed)

- We do **not** split α . The entire 0.05 goes in the right tail.
- The middle area to the right of zero is $0.5000 - 0.0500 = \mathbf{0.4500}$.



Visualizing the Tails: One-Tailed Test

When H_1 specifies a direction ($>$ or $<$), we place the **entire** risk (Significance Level, α) into just **one** tail.

Example: $\alpha = 0.05$ (Right-Tailed)

- We do **not** split α . The entire 0.05 goes in the right tail.
- The middle area to the right of zero is $0.5000 - 0.0500 = \mathbf{0.4500}$.
- Look up 0.4500 in the z-table $\rightarrow$ **Critical Value:** $+1.65$.

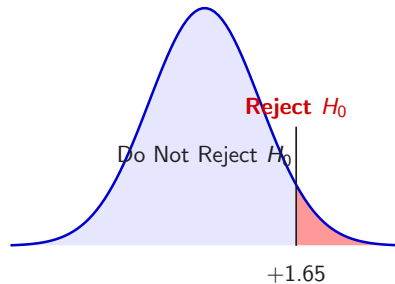


Visualizing the Tails: One-Tailed Test

When H_1 specifies a direction ($>$ or $<$), we place the **entire** risk (Significance Level, α) into just **one** tail.

Example: $\alpha = 0.05$ (Right-Tailed)

- We do **not** split α . The entire 0.05 goes in the right tail.
- The middle area to the right of zero is $0.5000 - 0.0500 = \mathbf{0.4500}$.
- Look up 0.4500 in the z-table $\rightarrow$ **Critical Value:** $+1.65$.



The p -Value in Hypothesis Testing

Moving Beyond "Reject / Do Not Reject"

So far, we have compared our computed z -statistic to a Critical Value. This only gives us a "Yes/No" or "Reject/Do Not Reject" answer.



Moving Beyond "Reject / Do Not Reject"

So far, we have compared our computed z -statistic to a Critical Value. This only gives us a "Yes/No" or "Reject/Do Not Reject" answer.

But what if management wants to know **how strong** the evidence is?

We use the **p -value**: The probability of observing a sample value as extreme as, or more extreme than, the value observed, assuming the null hypothesis is true.



Moving Beyond "Reject / Do Not Reject"

So far, we have compared our computed z -statistic to a Critical Value. This only gives us a "Yes/No" or "Reject/Do Not Reject" answer.

But what if management wants to know **how strong** the evidence is?

We use the **p -value**: The probability of observing a sample value as extreme as, or more extreme than, the value observed, assuming the null hypothesis is true.

The Universal Decision Rule:

Reject H_0 if the p -value $<$ Significance Level (α)



Interpreting the Weight of Evidence

The p -value allows us to grade the strength of our sample data.



Interpreting the Weight of Evidence

The p -value allows us to grade the strength of our sample data.

- If $p\text{-value} = 0.25$: The data provides **little to no evidence** that H_0 is false. We do not reject H_0 .



Interpreting the Weight of Evidence

The p -value allows us to grade the strength of our sample data.

- If $p\text{-value} = 0.25$: The data provides **little to no evidence** that H_0 is false. We do not reject H_0 .
- If $p\text{-value} = 0.04$: The data provides **moderate evidence** that H_0 is false. (We would reject H_0 at the 5% level, but not the 1% level).



Interpreting the Weight of Evidence

The p -value allows us to grade the strength of our sample data.

- If $p\text{-value} = 0.25$: The data provides **little to no evidence** that H_0 is false. We do not reject H_0 .
- If $p\text{-value} = 0.04$: The data provides **moderate evidence** that H_0 is false. (We would reject H_0 at the 5% level, but not the 1% level).
- If $p\text{-value} = 0.001$: The data provides **overwhelming evidence** that H_0 is false. It is virtually impossible this happened by chance.



Calculating the p -Value

Calculating the p -value is just finding the area under the normal curve!

For a One-Tailed Test:



Calculating the p -Value

Calculating the p -value is just finding the area under the normal curve!

For a One-Tailed Test:

- 1 Compute the z -statistic.



Calculating the p -Value

Calculating the p -value is just finding the area under the normal curve!

For a One-Tailed Test:

- 1 Compute the z -statistic.
- 2 Find the area between 0 and z in the standard normal table.



Calculating the p -Value

Calculating the p -value is just finding the area under the normal curve!

For a Two-Tailed Test:

For a One-Tailed Test:

- 1 Compute the z -statistic.
- 2 Find the area between 0 and z in the standard normal table.
- 3 Subtract that area from 0.5000 to find the area in the tail. **This is your p -value.**



Calculating the p -Value

Calculating the p -value is just finding the area under the normal curve!

For a Two-Tailed Test:

- 1 Compute the z -statistic.

For a One-Tailed Test:

- 1 Compute the z -statistic.
- 2 Find the area between 0 and z in the standard normal table.
- 3 Subtract that area from 0.5000 to find the area in the tail. **This is your p -value.**



Calculating the p -Value

Calculating the p -value is just finding the area under the normal curve!

For a One-Tailed Test:

- 1 Compute the z -statistic.
- 2 Find the area between 0 and z in the standard normal table.
- 3 Subtract that area from 0.5000 to find the area in the tail. **This is your p -value.**

For a Two-Tailed Test:

- 1 Compute the z -statistic.
- 2 Find the area between 0 and z .



Calculating the p -Value

Calculating the p -value is just finding the area under the normal curve!

For a One-Tailed Test:

- 1 Compute the z -statistic.
- 2 Find the area between 0 and z in the standard normal table.
- 3 Subtract that area from 0.5000 to find the area in the tail. **This is your p -value.**

For a Two-Tailed Test:

- 1 Compute the z -statistic.
- 2 Find the area between 0 and z .
- 3 Subtract from 0.5000 to find the area in *one* tail.



Calculating the p -Value

Calculating the p -value is just finding the area under the normal curve!

For a One-Tailed Test:

- 1 Compute the z -statistic.
- 2 Find the area between 0 and z in the standard normal table.
- 3 Subtract that area from 0.5000 to find the area in the tail. **This is your p -value.**

For a Two-Tailed Test:

- 1 Compute the z -statistic.
- 2 Find the area between 0 and z .
- 3 Subtract from 0.5000 to find the area in *one* tail.
- 4 **Multiply by 2.** (*Because H_1 did not specify a direction, the extreme value could have happened in either tail!*)



15 Minute Break

Fully Solved p -Value Examples

Example 1: Dole Pineapple (One-Tailed)

Textbook Example: Overfilled Cans

Dole is concerned that their 16-ounce can is being **overfilled** ($H_1 : \mu > 16$). The population standard deviation (σ) is **.03 ounce**.

A sample of **50 cans** ($n = 50$) showed a mean ($\bar{x}$) of **16.05 ounces**.

Task: Calculate the p -value. Does this provide overwhelming evidence against H_0 ?



Example 1: Dole Pineapple (One-Tailed)

Textbook Example: Overfilled Cans

Dole is concerned that their 16-ounce can is being **overfilled** ($H_1 : \mu > 16$). The population standard deviation (σ) is **.03 ounce**.

A sample of **50 cans** ($n = 50$) showed a mean ($\bar{x}$) of **16.05 ounces**.

Task: Calculate the p -value. Does this provide overwhelming evidence against H_0 ?

Step 1: Compute the z -statistic:

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{16.05 - 16.00}{0.03 / \sqrt{50}} = \frac{0.05}{0.00424} = \mathbf{11.79}$$



Example 1: Dole Pineapple (One-Tailed Solution)

Step 2: Find the Tail Area



Example 1: Dole Pineapple (One-Tailed Solution)

Step 2: Find the Tail Area

- Our computed $z = 11.79$.



Example 1: Dole Pineapple (One-Tailed Solution)

Step 2: Find the Tail Area

- Our computed $z = 11.79$.
- Look at your Standard Normal Table. It stops at $z = 3.09$ (which covers 0.4990 of the area).



Example 1: Dole Pineapple (One-Tailed Solution)

Step 2: Find the Tail Area

- Our computed $z = 11.79$.
- Look at your Standard Normal Table. It stops at $z = 3.09$ (which covers 0.4990 of the area).
- A z -score of 11.79 is completely off the charts!



Example 1: Dole Pineapple (One-Tailed Solution)

Step 2: Find the Tail Area

- Our computed $z = 11.79$.
- Look at your Standard Normal Table. It stops at $z = 3.09$ (which covers 0.4990 of the area).
- A z -score of 11.79 is completely off the charts!
- Therefore, the area between 0 and 11.79 is basically 0.5000.



Example 1: Dole Pineapple (One-Tailed Solution)

Step 2: Find the Tail Area

- Our computed $z = 11.79$.
- Look at your Standard Normal Table. It stops at $z = 3.09$ (which covers 0.4990 of the area).
- A z -score of 11.79 is completely off the charts!
- Therefore, the area between 0 and 11.79 is basically 0.5000.
- $p\text{-value} = 0.5000 - 0.5000 \approx \mathbf{0.0000}$.

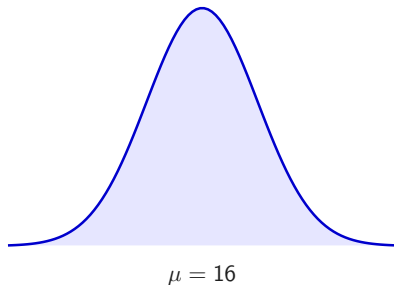


Example 1: Dole Pineapple (One-Tailed Solution)

Step 2: Find the Tail Area

- Our computed $z = 11.79$.
- Look at your Standard Normal Table. It stops at $z = 3.09$ (which covers 0.4990 of the area).
- A z -score of 11.79 is completely off the charts!
- Therefore, the area between 0 and 11.79 is basically 0.5000.
- $p\text{-value} = 0.5000 - 0.5000 \approx \mathbf{0.0000}$.

Because $p \approx 0.000$, we have **overwhelming** evidence to reject H_0 . It is statistically impossible to get a sample mean of 16.05



Our $z = 11.79$



Example 2: Crosset Trucking (Two-Tailed)

Textbook Exercise 5: Crosset Trucking

The manufacturer of the X-15 tire claims the mean mileage is **60,000 miles**. The population standard deviation (σ) is **5,000 miles**.

Crosset Trucking bought a sample of **48 tires** ($n = 48$) and found the mean mileage to be **59,500 miles**.

Task: Is Crosset's experience **different** from the claim? Calculate the p -value and make a decision at the $\alpha = 0.05$ level.



Example 2: Crosset Trucking (Two-Tailed)

Textbook Exercise 5: Crosset Trucking

The manufacturer of the X-15 tire claims the mean mileage is **60,000 miles**. The population standard deviation (σ) is **5,000 miles**.

Crosset Trucking bought a sample of **48 tires** ($n = 48$) and found the mean mileage to be **59,500 miles**.

Task: Is Crosset's experience **different** from the claim? Calculate the p -value and make a decision at the $\alpha = 0.05$ level.

Step 1: Hypotheses:

- $H_0 : \mu = 60,000$
- $H_1 : \mu \neq 60,000$ (**Two-Tailed!**)



Example 2: Crosset Trucking (z -statistic)

Step 2: Calculate the Test Statistic (z)

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$



Example 2: Crosset Trucking (z-statistic)

Step 2: Calculate the Test Statistic (z)

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$z = \frac{59,500 - 60,000}{5,000 / \sqrt{48}}$$



Example 2: Crosset Trucking (z-statistic)

Step 2: Calculate the Test Statistic (z)

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$z = \frac{59,500 - 60,000}{5,000 / \sqrt{48}}$$

$$z = \frac{-500}{5,000 / 6.928} = \frac{-500}{721.68} = \mathbf{-0.693 \approx -0.69}$$



Example 2: Crosset Trucking (z-statistic)

Step 2: Calculate the Test Statistic (z)

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$z = \frac{59,500 - 60,000}{5,000 / \sqrt{48}}$$

$$z = \frac{-500}{5,000 / 6.928} = \frac{-500}{721.68} = \mathbf{-0.693 \approx -0.69}$$

Our sample mean is only 0.69 standard deviations below the claimed mean. Let's find the exact probability of this happening.



Example 2: Crosset Trucking (p -Value Solution)

Step 3: Calculate the Two-Tailed p -Value



Example 2: Crosset Trucking (p -Value Solution)

Step 3: Calculate the Two-Tailed p -Value

- Look up $z = 0.69$ in the table. The area between 0 and 0.69 is **0.2549**.



Example 2: Crosset Trucking (p -Value Solution)

Step 3: Calculate the Two-Tailed p -Value

- Look up $z = 0.69$ in the table. The area between 0 and 0.69 is **0.2549**.
- Find the single tail area:
 $0.5000 - 0.2549 = \mathbf{0.2451}$.



Example 2: Crosset Trucking (p -Value Solution)

Step 3: Calculate the Two-Tailed p -Value

- Look up $z = 0.69$ in the table. The area between 0 and 0.69 is **0.2549**.
- Find the single tail area:
 $0.5000 - 0.2549 = \mathbf{0.2451}$.
- **Multiply by 2!** (Because H_1 was $\neq$, the extreme result could have been in either tail).



Example 2: Crosset Trucking (p -Value Solution)

Step 3: Calculate the Two-Tailed p -Value

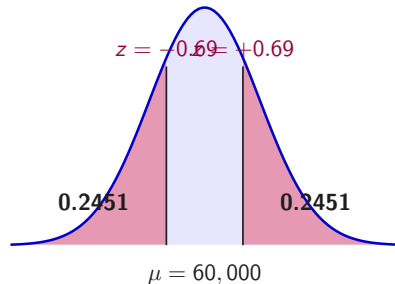
- Look up $z = 0.69$ in the table. The area between 0 and 0.69 is **0.2549**.
- Find the single tail area:
 $0.5000 - 0.2549 = \mathbf{0.2451}$.
- **Multiply by 2!** (Because H_1 was $\neq$, the extreme result could have been in either tail).
- $p\text{-value} = 2 \times 0.2451 = \mathbf{0.4902}$.



Example 2: Crosset Trucking (p -Value Solution)

Step 3: Calculate the Two-Tailed p -Value

- Look up $z = 0.69$ in the table. The area between 0 and 0.69 is **0.2549**.
- Find the single tail area:
 $0.5000 - 0.2549 = \mathbf{0.2451}$.
- **Multiply by 2!** (Because H_1 was $\neq$, the extreme result could have been in either tail).
- $p\text{-value} = 2 \times 0.2451 = \mathbf{0.4902}$.



Decision: 0.4902 is much greater than $\alpha = 0.05$. **Do not reject** H_0 . There is a 49% chance this difference is just sampling error.



Example 3: Live Class Activity

Textbook Exercise 21 (Adapted)

A university claims the mean starting salary of its graduates is **\$45,000**.

A random sample of **120 graduates** ($n = 120$) has a mean salary ($\bar{x}$) of **\$45,500**. The population standard deviation (σ) is known to be **\$3,000**.

Task: Does this evidence prove the true mean salary is **not** \$45,000? Use a **10% significance level** ($\alpha = 0.10$).

Group Task: Take 5 minutes. Formulate the hypotheses, find the z-statistic, compute the exact p-value, and state your final business decision!



Example 3: Live Class Solution

1. **Hypotheses:** $H_0 : \mu = 45,000$ vs. $H_1 : \mu \neq 45,000$ (Two-Tailed)



Example 3: Live Class Solution

1. **Hypotheses:** $H_0 : \mu = 45,000$ vs. $H_1 : \mu \neq 45,000$ (Two-Tailed)
2. **Calculate z-statistic:**

$$z = \frac{45,500 - 45,000}{3,000/\sqrt{120}}$$

$$z = \frac{500}{273.86} = \mathbf{1.83}$$



Example 3: Live Class Solution

1. **Hypotheses:** $H_0 : \mu = 45,000$ vs. $H_1 : \mu \neq 45,000$ (Two-Tailed)

2. **Calculate z-statistic:**

$$z = \frac{45,500 - 45,000}{3,000/\sqrt{120}}$$

$$z = \frac{500}{273.86} = \mathbf{1.83}$$

3. **Calculate p-value:**

- Area for $z = 1.83$ is **0.4664**.
- Tail = $0.5000 - 0.4664 = \mathbf{0.0336}$.
- $p\text{-value} = 2 \times 0.0336 = \mathbf{0.0672}$.



Example 3: Live Class Solution

1. **Hypotheses:** $H_0 : \mu = 45,000$ vs. $H_1 : \mu \neq 45,000$ (Two-Tailed)

2. **Calculate z-statistic:**

$$z = \frac{45,500 - 45,000}{3,000/\sqrt{120}}$$

$$z = \frac{500}{273.86} = \mathbf{1.83}$$

3. **Calculate p-value:**

- Area for $z = 1.83$ is **0.4664**.
- Tail = $0.5000 - 0.4664 = \mathbf{0.0336}$.
- $p\text{-value} = 2 \times 0.0336 = \mathbf{0.0672}$.

4. **Decision Rule:**

- Our α is **0.10**.
- Our $p\text{-value}$ is **0.0672**.

Because the $p\text{-value}$ (0.0672) is **less than** α (0.10), we **Reject** H_0 .

We can conclude the true mean salary is not \$45,000.

